

# Low-temperature behavior of the Casimir-Polder free energy and entropy for an atom interacting with graphene

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## Abstract

© 2018 American Physical Society. The analytic expressions for the free energy and entropy of the Casimir-Polder interaction between a polarizable and magnetizable atom and a graphene sheet are found in the limiting case of low temperature. In so doing, the response of graphene to electromagnetic fluctuations is described in the framework of the Dirac model by means of the polarization tensor in (2+1)-dimensional space-time. It is shown that the dominant contribution to the low-temperature behavior is given by an explicit dependence of the polarization tensor on temperature as a parameter. We demonstrate that the Lifshitz theory of atom-graphene interaction satisfies the Nernst heat theorem, i.e., is thermodynamically consistent. On this basis possible reasons of thermodynamic inconsistency arising for the Casimir-Polder and Casimir interactions in the case of Drude metals are discussed. The conclusion is made that, although large thermal effect arising in the Casimir interaction between Drude metals at short separations should be considered as an artifact, the giant thermal effect predicted for graphene systems is an important physical phenomenon which awaits its experimental observation.

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