

# Construction of a representative model based on computed tomography

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## Abstract

© PNRPU. The simulation of the stress-strain state of porous or multiphase media is an important task nowadays. The application of the mathematical model of continuum mechanics to such media will make it possible to extend the scope of the problems to be solved. The development of nondestructive methods of control, such as computed tomography, allows obtaining data on the structure of various heterogeneous materials. This task is especially important in the areas of clinical medicine and biology. The paper presents the method aimed at determining mechanical properties of a representative element using computed tomography. Based on the finite element method for a given region, a finite-element ensemble is constructed using the scanning data on a computer tomograph of a real sample. For the obtained sample, numerical experiments are performed in the kinematic formulation, after which the problem of the stress-strain state is solved. The stresses obtained as a result of the calculations are averaged and used to determine the components of the elastic constant tensor. Thus, the anisotropic properties of the representative element are determined. To determine the orthotropic properties of the representative element, a target function is introduced, the arguments of which are unknown directions of orthotropy. These unknown directions are determined from the condition of minimizing the objective function. The transformation of the rotation to an anisotropic matrix of elastic constants makes it possible to determine the components of the elastic constant tensor in the orthotropic axes. As an illustration of the technique, calculations of a porous sample are given in the paper, and the obtained results are evaluated. For the quantitative comparison, the invariant of the stress tensor is used. The obtained results illustrate not only a sufficient accuracy of describing the medium in terms of continuity, but also a discrepancy in the results in the case of large porosity.

<http://dx.doi.org/10.15593/perm.mech/2018.3.10>

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## Keywords

Anisotropy, Computed tomography, Finite element method, Orthotropic axes, Orthotropy, Representative element

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