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Development and Research of the Mathematical Model of Planar Motion of a Vehicle with a Semitrailer

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Abstract—On horizontal nondeformable ground, we consider the movement of a road train consisting of a biaxial tractor car and triaxial semitrailer treated as solid bodies. Based on the Lagrange's equations of the second kind, we develop a nonlinear mathematical model of its plane motion, using the position of the fifth-wheel coupling and rotating angles of the tractor and semitrailer body as generalized coordinates. We analyze and linearize the constructed system of equations and obtain a linear mathematical model describing the small lateral displacements and rotations of the elements of a road train when it is moving at a high longitudinal speed, small jackknifing angle, and small rotation angle of the steering wheels. Using the equivalent transformations of the obtained system of equations, we construct a state-space linear model of the lateral motion of the road train. A comparative analysis of the use of linear and nonlinear models to describe the road train's motion, carrying out standard maneuvers, is performed. It is shown that, if the restrictions are satisfied, then the results of nonlinear and linear model usage are quite close to each other and sufficiently well agree with the results of the field tests. The developed model, unlike the already known ones, is fairly simple (linear). Further, it could be used for an analytical synthesis of the control laws for the lateral component of the motion of road trains.

Keywords: road train, planar motion, Lagrange's equations of the second kind, nonlinear model, lateral motion, linear model, analysis

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1. INTRODUCTION

The active use of computer simulation methods at all stages of the lifecycle of a manufactured product is an objective trend in the development of the contemporary automotive industry. Various models describing various car characteristics with varying degrees of precision are necessary, depending on the specificity of problems to be solved. For example, to find aerodynamic characteristics, we have to describe the geometry of the body of the investigated car, to estimate the fuel-economy properties, we need a detailed description of the engine and transmission characteristics, and to investigate the vibroload, we have to represent the car as a multimass oscillating system.

At the moment, many models used to investigate various processes taking place during the movement of a car have been developed and described (see [1]). However, the majority of them are intended to investigate biaxial or triaxial vehicles (multiaxial ones occur less frequently). A separate important and complex problem is the investigation of the specificities of the movement of concatenated motor vehicles, which are cars with one or more trailers or semitrailers (road trains). This is caused by the active use of road trains due to their high cost efficiency for cargo traffic: according to industry experts, the application of road trains allows us to reduce the manufacturing cost of long-distance and medium-distance cargo traffic by 30% (see [2]).

Various approaches to modeling the movement of road trains are proposed in [3–7]. The main disadvantages of the provided approaches and models developed on their base are their high dimensions, fundamental nonlinearity, and complexities of accounting for the reaction forces in the coupling of the fifth wheel. Note that the possibility to linearize the problem is crucially important for various engineering