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Rigorous Formulation of the Lasing Eigenvalue Problem as a Spectral Problem for a Fredholm Operator Function

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Abstract—We propose a new convenient for mathematical investigation formulation of the lasing eigenvalue problem as a spectral problem for an operator-valued function, which involves boundary integral operators. We prove that these integral operators are weakly singular and the operator of the problem is Fredholm with index zero.

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1. INTRODUCTION

Various two-dimensional (2D) microcavity lasers have been investigated numerically with the aid of a modified electromagnetic eigenvalue problem, specifically tailored to extract the threshold values of gain in addition to the emission frequencies (see, for example, [15, 18, 19], and references therein). Such a modified formulation called the Lasing Eigenvalue Problem (LEP) was first introduced in 2004 in [11] and since then has gained credit in the photonics community. The greatest progress may have been achieved for two-dimensional microcavities with uniform gain in [13], where the original problem was reduced equivalently to a nonlinear spectral problem for the system of Muller boundary integral equations (BIEs), which was solved accurately by the Nystrom method. Derived first by Muller [10] this system has become a reliable and efficient tool for analysis of the electromagnetic field in the presence of a 2D homogeneous dielectric object with an arbitrary smooth boundary. Particularly, Muller BIEs were used for computations of eigenmodes of fully active [13] and passive microcavities [2, 3]. The original problem for microcavities with active regions have been also reduced recently to the system of Muller BIEs [14]. Numerical and theoretical investigations of microcavities with active regions are very important [12], but such studies have not been carried out in sufficient detail by rigorous mathematical methods.

In this paper we propose a new formulation of LEP for microcavities with active regions as a nonlinear spectral problem for a fredholm operator-valued function, which involves boundary integral operators. In Section 2 we describe the nonlinear spectral problem for the system of Muller BIEs constructed in [14]. In Section 3 we prove that all the boundary integral operators are weakly singular or have smooth kernels (Lemmas 1–4). It follows from these lemmas that the operator of the problem has the form $I - B$, where the operator B is compact (Theorem 1, Section 4) and I is the identical operator in the space of continuous functions. Obtained formulation is convenient for future study of the problem on the base of fundamental results of the theory of operator-valued functions in a pair of Banach spaces (see, for

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