

Application of Riemann–Hilbert Problem Solutions to a Study of Nonlinear Boundary Value Problems for Timoshenko Type Inhomogeneous Shells with Free Edges

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Abstract—The paper deals with the study of solvability of geometrically nonlinear boundary value problem for elastic shallow isotropic inhomogeneous shells with free edges within S. P. Timoshenko shear model. The problem is reduced to one nonlinear equation whose solvability is proved with the use of contracting mappings principle.

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INTRODUCTION

The solvability of nonlinear problems for thin elastic shells is currently sufficiently fully studied in the framework of the simplest Kirchhoff–Love model (see [1–6] and the literature quoted in them). The questions of existence of solutions of nonlinear problems within the more general models of the theory of the shells, not based on hypotheses of Kirchhoff–Love were included into the known list of unresolved problems of the mathematical theory of shells of academician I.I. Vorovich ([1], p. 349) and until recently they remained open. Today there is a number of works [7–12] in which nonlinear problems are studied within the shear model of S.P. Timoshenko. The method of studies in [7–12] is based on integral representations for the generalized displacements containing the arbitrary holomorphic functions. The holomorphic functions are defined so that the generalized displacements satisfy the given boundary conditions. Construction of integral representations is one of the essential moments of a method of studies. For their construction two approaches are used. The first approach is based on application of solutions of a problem of Riemann–Hilbert for the holomorphic functions in a unit disk. Therefore the flat domain which is homeomorphic to middle surface of a shell, or is supposed from the very beginning by a unit disk [7–9, 11], or conformally mapped onto a unit disk [10]. The second approach for determine the holomorphic functions uses the theory of integrals of Cauchy type with the real density. These densities are defined as the solutions of system of one-dimensional singular integral equations [12]. In all these works [7–12] shell was assumed by the homogeneous. In the present work the first approach develops for case of the inhomogeneous shells of Timoshenko type with the free edges, the middle surface of which is a homeomorphic to a unit disk. The main difference of the offered studies from [7–12] is what in this case it is necessary to deal with system of differential equations with variable coefficients what significantly complicates a obtaining of integral representations for the generalized displacements satisfying the given boundary conditions.

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