ISSN 1995-0802, Lobachevskii Journal of Mathematics, 2018, Vol. 39, No. 6, pp. 847–854. © Pleiades Publishing, Ltd., 2018.

A Cutting Method with Approximation of a Constraint Region and an Epigraph for Solving Conditional Minimization Problems

O. N. Shulgina[1*], R. S. Yarullin[1], and I. Ya. Zabotin[1***]**

(Submitted by A. M. Elizarov)

[1] *Kazan (Volga region) Federal University, ul. Kremlevskaya 18, Kazan, 420008 Russia*

Received July 12, 2017

Abstract—We propose a method which belongs to a class of cutting methods for solving a convex programming problem. The developed method differs from traditional algorithms of the mentioned class by the following fact. This method uses approximation of the feasible set and the epigraph of the objective function simultaneously for the solving problem. In the method cutting planes are constructed by subgradients of the objective function and functions which define the constrained set. In this case while initial approximating sets are chosen as polyhedral sets, each iteration point is found by solving a linear programming problem independently of the type of functions which define the solving problem. Moreover, unlike most the cutting methods the proposed method is characterized by possibility of updating approximating sets due to dropping accumulating constraints.

DOI: 10.1134/S1995080218060197

Keywords and phrases: *An approximating set, conditional minimization, convergence, a cutting plane, an epigraph, a sequence of approximations.*

Cutting methods are used for solving convex programming problems quite often when the objective function is linear or the constraint set is determined by linear functions. The class of these methods is quite wide (e.g., [1–9]). In these methods to find iteration points it is applied sequential approximation of either the constraint set or the epigraph of the objective function depending on the type of functions which define the initial problem.

The cutting method which is proposed in this paper approximates both the constraint set and the epigraph by polyhedral sets. During constructing iteration points this feature allows to solve an auxiliary linear programming problems at each step independently of types of the constraint functions and the objective function.

Further, amount of cutting planes which determine approximating sets is usually increased unboundedly under incrementing the number of iterations. Considering this fact complexity of solving problems for finding iteration points is increased from step to step. Moreover, the feature of the proposed method is as follows that it allows to update approximating sets by periodically dropping any cutting planes constructed in the minimization process. The approach for constructing cutting algorithms with updating embedding sets was developed and used by the authors in [10–12].

Let $f(x)$, $F(x)$ be convex functions defined in an n-dimensional Euclidian space R_n. Suppose that $D' = \{x \in \mathrm{R}_n : F(x) \leq 0\}$, $D'' \subset \mathrm{R}_n$ is a convex closed set, $D = D' \bigcap D''$, and the function $f(x)$ achieves its minimum value on the set D. Solve the following optimization problem:

$$\min\{f(x) : x \in D\}. \tag{1}$$

[*] E-mail: ONShul@mail.ru
[**] E-mail: YarullinRS@gmail.com
[***] E-mail: IYaZabotin@mail.ru