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Generalized Reduced Module of a Domain Over the Unit Disc with Circular and Radial Slits

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Abstract—For $(n+1)$-ly connected planar domain D with analytic boundary we construct the function $F(w, w_0) = (w - w_0)f(w, w_0)$ which maps D conformally onto the unit disk with circular and radial slits. We show that if $n \geq 2$, then Mityuk's function, $M(w) = -(2\pi)^{-1} \ln |f(w, w)|$, representing the generalized reduced module of the domain D has at least one stationary point in D.

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1. INTRODUCTION

Classical task in the complex function theory concerns with the construction of the conformal mappings

$$F(w, w_0) = (w - w_0)f(w, w_0), \quad f(w_0, w_0) \neq 0, \tag{1}$$

from the planar finitely connected regions onto the canonical domains such as the unit disk centered at the origin with cuts along the arcs of prescribed form, namely, circular concentric arcs, radial slits, or their various disjoint combinations.

I. P. Mityuk [1] has proposed a way to define a generalized reduced modules connected with function $F(w, w_0)$. The generalized reduced module,

$$M(w) = -\frac{1}{2\pi} \ln |f'(w, w)| \tag{2}$$

of a multiply connected domain D at a point w will be called *Mityuk's function* with respect to the distinguished canonical domain.

Connection of the functions (2) with the exterior inverse boundary value problems goes back to Gakhov [2]. As it has appeared, the non-emptiness of the critical points set of the function $M(w)$ is equivalent to the suitable exterior problem. The existence of critical points of Mityuk's function in the case of circular concentric slits has been proved by Kinder [3]. The case of circular and radial slits is studied in the present report (see also [4] and [5]).

Let D be $(n+1)$-ly connected domain with the boundary ∂D, consisting of disjoint analytic curves L_k, $k = \overline{0, n}$; the contour L_0 encircles the others. In the section 1 we shell define the auxiliary functions involving in the construction of the mapping (1) of the domain D onto the unit disk with radial and circular slits. In the Section 2 the existence and the univalence of such a mapping will be proved. In the Section 3 we show that the function (2) has at least one stationary point in D if $n \geq 2$; the doubly-connected example is constructed where the function (2) has no stationary points.

Let us note that a number of the assertions of this note can be transferred to the general case of Jordan domains; we won't stop on details.

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