Boundary-Value Problems for the Helmholtz Equation for a Half-Plane with a Lipschitz Inclusion

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Abstract—I consider the problems of diffraction of electromagnetic waves on a half-plane, which has a finite inclusion in the form of a Lipschitz curve. Boundary value problems, modeling the process of wave diffraction, are constructed in the form of Helmholtz equations and boundary conditions on the boundary, formulated in terms of traces, as well as the radiation conditions at infinity. I carry out research on these problems in generalized Sobolev spaces. I proved the solvability of the boundary value problems of Dirichlet and Neumann. I have obtained solutions of boundary value problems in the form of functions that by their properties are analogs of the classical potentials of single and double layers. Boundary problems are reduced to integral equations of the second kind.

1. INTRODUCTION

Since 1970, boundary problems on Lipschitz domains have been formed in a special field of research. This is caused both with applications in electrodynamics and other fields, and with the theoretical importance of these studies. An extensive bibliography on this range of issues and the most important results are given in the articles M.S. Agranovich [1−3], M. Costabel [4], D. Jerison and C. Kenig [5−7], M. Mitrea and M. Taylor [8, 9]. A detailed summary of the main results of the theory of boundary value problems on Lipschitz domains can be found in [1, 10, 11]. An essential part of this theory is devoted to boundary integral operators. A summary of the theory of integral operators on Lipschitz domains is contained in article [4]. Note that this work is the most cited in this field. An important property of the Lipschitz domain is the possibility approximations by infinitely differentiable domains from either side boundaries of this domain (see [1, 2]). Lipschitz domains also satisfy the condition of a uniform cone (see [1]). These properties are used in the present paper to investigate the solvability of boundary value problems of diffraction on an unbounded boundary with a Lipschitz inclusion.

In the papers [13−15] we investigated the boundary value problems for the Helmholtz equation in domains with an rough smooth and piecewise smooth boundaries. These studies are based on the use of generalized potentials of single and double layers. In contrast to the classical potentials defined on closed domains, generalized potentials are considered on open curves and on domains with an infinite boundary. In the present paper this technique is extended to the case of a half-plane with a Lipschitz inclusion of finite size.

In the study of boundary value problems on Lipschitz boundaries, operators of potential type are introduced, which are analogues of operators of single and double layers (e.g., see [3]). These operators have properties close to those of the classical potentials of single and double layers, which makes it possible to apply, after necessary refinements, the same reasoning technique as in the classical case (e.g., see [3, 4]).

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