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Uzawa-type Iterative Solution Methods for Constrained Saddle Point Problems

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Abstract—For finite-dimensional saddle point problem with a nonlinear monotone operator and constraints on direct variables, iterative methods are developed, which in the potential case can be viewed as preconditioned Uzawa methods or as Uzawa-block relaxation methods. Convergence conditions of the iterative methods are formulated in the form of operator inequalities connecting the operator of the problem and the preconditioning matrix. When applied to mesh problems, this allows us to construct suitable preconditioners that ensure the convergence and effective implementation of iterative methods and to obtain the admissible intervals of iterative parameters which don't depend on mesh parameters. The presented results are based on the general theory developed by the author with co-authors in recent years.

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INTRODUCTION

We construct and investigate the iterative solution methods for finite-dimensional saddle point problem

$$\begin{pmatrix} A & -B^T \\ -B & 0 \end{pmatrix} \begin{pmatrix} x \\ \lambda \end{pmatrix} + \begin{pmatrix} \partial\Psi(x) \\ 0 \end{pmatrix} \ni \begin{pmatrix} f \\ -g \end{pmatrix} \qquad (*)$$

with a strongly monotone operator A, matrix B of full column rank, and maximal monotone operator $\partial\Psi$ (subdifferential of a convex function Ψ). In what follows we assume that problem (*) is a mesh approximation of one of the following problem:

—variational inequality with strongly monotone differential operator and constraints on the gradient of the solution;

—state constrained optimal control problem governed by a linear partial differential equation.

The actual methods for solving variational inequalities with constraints on the gradient of the solution, based on the application of the augmented Lagrangian, have been studied in detail (see the books [1–4]). In numerous papers iterative methods of solving optimal control problems with constraints on the state of the system are constructed and investigated [5–10] (see also the bibliography of these articles).

Iterative methods for linear saddle point problems, corresponding to (*) with linear A and $\Psi = 0$, are thoroughly investigated (see monograph [11], survey article [12] and the bibliography therein). The theory of iterative methods for constrained saddle point problems was not deeply developed. In this

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