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Confidence Bounds and Narrowest Reliable Intervals in D-Posterior Approach

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Abstract—Several new methods of confidence and of asymptotically confidence limits in the d-posterior approach is proposed. For the so-called reliable two-sided intervals, close in construction to Bayesian intervals, the method of constructing the narrowest intervals is given.

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1. INTRODUCTION

In the article [1] the concept of confidence families of subsets of a parametric space in the framework of a d-posterior approach was proposed. In this approach, the quality of a statistical procedure is associated with a conditional distribution of output parameter by a fixed decision—a fixed result of the procedure. The definition of a confidence family is structured such that, as well as in the classical Neyman–Pearson approach, a relationship is maintained between the confidence statement and the solution of the guarantee hypothesis testing.

Suppose that we observe a sample $\xi^{(n)} = (\xi_1, \ldots, \xi_n)$ following from a distribution with density $f_\theta(x)$, $x \in \mathbf{X}$, with respect to some σ-finite measure μ on a measurable sample space $(\mathbf{X}, \mathcal{X})$. The unknown parameter θ is the realization of a random variable ϑ with a known prior distribution G on the measurable space $(\Theta, \mathcal{B})$. According to the experimental data $\xi^{(n)} = x^{(n)}$ it is required to test the hypothesis $H_0 : \theta \in \Theta_0 \subset \Theta$ vs. $H_1 : \theta \in \Theta_1 = \Theta_0^c$. Denote $\mathbf{P}$ the joint distribution of the vector $(\vartheta, \xi^{(n)})$:

$$\mathbf{P}\{\vartheta \in B, \xi^{(n)} \in Y\} := \int\limits_B \left(\int\limits_Y \prod_{k=1}^n f_\theta(x_k)\mu(dx_k) \right) G(d\theta)$$

for $B \in \mathcal{B}, Y \in \otimes_{k=1}^n \mathcal{X}$. The conditional distribution $\mathbf{P}_\theta$ of the observation vector by a fixed value of the parameter θ is calculated through the density $f_\theta(x^{(n)}) = \prod_1^n f_\theta(x_k)$ with respect to the direct product measure $\otimes_{k=1}^n \mu$ on $\otimes_{k=1}^n \mathcal{X}$.

The decision rule $\delta = \delta(x^{(n)})$, which takes the solution d_0 in favor of H_0 or the decision d_1 in favor of H_1, must guarantee given restriction β_0 to conditional error probability (d-risk) of the first kind

$$R(d_0; \delta) := \mathbf{P}\{\vartheta \notin \Theta_0 | \delta = d_0\} \leqslant \beta_0.$$

Among similar decision rules it is required to find rule (the optimal rule) that minimizes the risk of the second kind

$$R(d_1; \delta) := \mathbf{P}\{\vartheta \in \Theta_0 | \delta = d_1\}.$$

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