

# Periodic Conjugation Problem for Linear Elliptic Equations of Second Order with Constant Coefficients

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**Abstract**—We consider a periodic problem of conjugation on the real axis for a linear differential elliptic equation of second order with constant coefficients. There is known a representation of general solution of the equation in terms of analytic functions depending on affine connected variables. We introduce auxiliary analytic functions enabling us to reduce the problem to a system of two periodic Riemann boundary value problems. That problem was solved first by L. I. Chibrikova. We use her results for solving of our problem. We obtain its explicit solution and conditions of solvability, evaluate the index and defect numbers.

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## 1. INTRODUCTION

In recent years, we began to study the periodic boundary value problems for linear partial differential equations of elliptic type, associated with the basic equations of a plane isotropic elasticity theory (see, for instance, [1, 2]). In the present paper we research a boundary value problem of conjugation for elliptic equations associated with the basic equations of plane anisotropic elasticity theory.

Let  $L = \sum_{k=0}^2 a_k \frac{\partial^2}{\partial x^{2-k} \partial y^k}$  be an elliptic operator with constant complex coefficients  $a_0, a_1, a_2$ . The operator ellipticity implies that both  $a_2 \neq 0$  and the characteristic polynomial  $a_0 + a_1 s + a_2 s^2$  possesses no real roots. We denote the roots of this polynomial by  $s_1$  and  $s_2$ . We also assume these roots to be different and that  $\Im s_q > 0, q = 1, 2$ . Consider the equation

$$Lf = 0 \tag{1}$$

in the domain  $D \subset \mathbb{C}$ .

Without loss of generality we assume that the origin belongs to  $D$ .

## 2. ONE-PERIODIC SOLUTIONS OF EQUATION (1)

Let  $\omega$  be a complex nonzero number. We study the  $\omega$ -periodic solutions of equation (1). The general solution of equation (1) in the domain  $D$  takes (among others) the form [3]

$$f(z) = \varphi_1(z_1) + \varphi_2(z_2), \tag{2}$$

here  $z_q = T_q(z) := x + s_q y, q = 1, 2$ , is an orientation preserving affine transform of the complex variable  $z = x + iy$  plane, the functions  $\varphi_q$  are analytic in the domains  $D_q := T_q(D), q = 1, 2$ . Put  $\omega_1 = T_1(\omega), \omega_2 = T_2(\omega)$ . Representation (2) is unique under the following additional assumption:  $\varphi_2(0) = 0$ . We

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