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Euclidean maximum moduli of plane domains and their applications

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Abstract

© 2018, © 2018 Informa UK Limited, trading as Taylor & Francis Group. For a plane domain we study correlations of the Euclidean maximum modulus and three hyperbolic domain characteristics connected with the Poincaré metric of the domain and the distance function. We prove that the Laplacian of the hyperbolic radius of every domain of hyperbolic type is a subharmonic function. Also, for any doubly connected domain we prove asymptotically sharp estimates for the hyperbolic characteristics using the Euclidean maximum modulus of the domain. In addition, we obtain applications of these estimates to Hardy and Schwarz–Pick type inequalities.

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Keywords

Conformal modulus, Hardy type inequality, Poincaré metric, Primary 30F45, Schwarzian, Secondary 30C20

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