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Letter

On Kasner solution in Bianchi I $f(T)$ cosmology

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Abstract Recently the cosmological dynamics of an anisotropic Universe in $f(T)$ gravity became an area of intense investigations. Some earlier papers devoted to this issue contain contradictory claims about the nature and propertied of vacuum solutions in this theory. The goal of the present paper is to clarify this situation. We compare properties of $f(T)$ and $f(R)$ vacuum solutions and outline differences between them. The Kasner solution appears to be an exact solution for the $T = 0$ branch, and an asymptotic solution for the $T \neq 0$ branch. It is shown that the Kasner solution is a past attractor if $T < 0$, being a past and future attractor for the $T > 0$ branch.

The Kasner solution, being one of the first known exact solutions in relativistic cosmology [1] continues to be one of the most important exact solutions in general relativity (GR) or its modifications. One of the reasons is that despite this being a vacuum solution, it is a good approximation near a cosmological singularity for almost all matter sources (except for a stiff fluid) in a flat anisotropic Universe. Moreover, a general cosmological singularity is believed to be constructed as an infinite series of consecutive epochs, each of them being a particular Kasner solution with a good accuracy (though a mathematical proof of this scenario is still absent in full details; see, for example, [2])—the famous Belinskii–Khalatnikov–Lifshitz (BKL) scenario [3]. Therefore the Kasner set of solutions provides "building blocks" for the BKL picture.

If we assume that GR needs some modifications at UV scale, it is natural to expect that such modifications should change the behavior near a cosmological singularity significantly. That is why the fate of the Kasner solution in modified gravity theories is an area of intense investigations. A lot of efforts have been devoted to Kasner solutions and their modifications in quadratic gravity. We remind the reader that the Kasner solution is a solution for an anisotropically expanding Universe with scale factors changing as powers of time. These power exponents are subject of two conditions giving us their sum as well as the sum of their squares (both sums are equal to unity). In quadratic gravity, two different situations were identified:

- If the equations of motion are of the second order, as in GR (that is, in Gauss–Bonnet gravity), the power-law solution for the scale factor is an asymptotic solution. In the high-curvature regime, these two conditions for power exponents are different from those in the GR Kasner solution [4–6], while the GR Kasner solution is an asymptotic solution in the low-curvature regime.
- In fourth order gravity (like $R+R^2$ or a general quadratic gravity) the Kasner solution (with the same conditions for the exponents) is an *exact* vacuum solution. However, since the phase space has two additional dimensions in comparison with GR, a Kasner solution in quadratic gravity may be in some situations unstable [7,8].

Recently a new class of modified gravity theories has started to attract much attention. It is based on the Teleparallel Equivalent to General Relativity (TEGR)—a theory first considered by Einstein in the 1920s [9–11] where the Levi-Civita connection (torsion-free, non-zero curvature) has been replaced by Weitzenböck connection [12] (curvature-free, non-zero torsion), and curvature scalar R in the action by the torsion scalar T. It appears that despite different mathematical backgrounds, TEGR and GR are identical at the level of the equations of motion. For a review see, for example, the book [13]. Now it is well known that in cases more complicated than the standard Einstein–Hilbert action, the theory based on torsion has the equations of motion different from the theory based on curvature. In particular, $f(T)$ theory is not equivalent to $f(R)$ theory, if the function f is not a linear function.

This motivated studies of cosmological dynamics in $f(T)$ gravity. Recently many papers on this topic have appeared, mostly concentrating on FRW cosmology (see, for example,

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