

# ASYMPTOTIC EXPANSION OF POSTERIOR DISTRIBUTION OF PARAMETER CENTERED BY A $\sqrt{n}$ -CONSISTENT ESTIMATE

A. A. Zaikin\*

UDC 519.2

*The paper studies asymptotic behavior of posterior distribution of a real parameter centered by a  $\sqrt{n}$ -consistent estimate. The uniform analog of the Bernstein–von Mises theorem is proved. This result is extended to asymptotic expansion of the posterior distribution in powers of  $n^{-1/2}$ . This expansion is generalized as the expansion of expectations of functions with polynomial majorant with respect to posterior distribution. Bibliography: 14 titles.*

## 1. INTRODUCTION

One of the earliest results on asymptotic behavior of a posterior distribution is the Bernstein–von Mises theorem, which states asymptotic normality of the posterior distribution (see, for example, [12, p. 141]). A refinement of this result in the form of asymptotic expansions was obtained by Johnson in [7], Gusev in [2] and [3], Weng in [13], and by the author of the present paper in [14]. All of these asymptotics allow to choose different centerings of the variable in question, different parameters of the asymptotic distribution, and different terms in the expansions.

Important characteristics of such an expansion are determined by stochastic properties of the residual expansion. A notable result is given by Ghosh in [1], where he proved the uniformity of Johnson's expansion [7] with respect to the distribution parameter.

The asymptotic normality of posterior distribution is essentially based on the local asymptotic normality of the likelihood function and related concept of asymptotically sufficient statistics (see, for example, [11]). So far, only the asymptotic sufficiency of maximal likelihood estimate and differential sufficiency of the score function have been used in studying asymptotics of posterior distribution. However, the property of asymptotic sufficiency holds for a much broader class of statistics. In [8], Le Cam established asymptotic sufficiency of any  $\sqrt{n}$ -consistent estimate of the distribution parameter (the definition of the  $\sqrt{n}$ -consistent estimate is given in Sec. 2), where  $n$  is the sample size.

In the present paper, we generalize earlier results on asymptotic expansions of posterior distributions by centering the parameter with the help of a  $\sqrt{n}$ -consistent estimate. Namely, in papers [1, 7] and [3], expansions of posterior distribution of parameter centered by a maximum likelihood estimate are constructed; in paper [3], centering with the help of a Bayesian estimate is also considered. In earlier paper of the author [14], centering is performed by a fixed point of parameter space. Thus, centering with the help of a  $\sqrt{n}$ -consistent estimate includes all previous results and generalizes them essentially.

Another feature of our result is that it is uniform with respect to different values of the parameter. To the best of our knowledge, a result of this kind was obtained in paper [1] only.

The paper is organized as follows.

In Sec. 2, the notation is introduced. Then we present some assumptions on the asymptotic normality of posterior distribution and use them to prove intermediate results.

In Sec. 3, an analog of the Bernstein–von Mises theorem for centering with the help of a  $\sqrt{n}$ -consistent estimate  $T_n$  is stated (Theorem 1). In other words, the conditional distribution

---

\*Kazan Federal University, Kazan, Russia, e-mail: kaskrin@gmail.com.