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On Direct and Inverse Problems of Logarithmic Potential With Finite Number of Parameters

N. R. Abubakirov* **and L. A. Aksent'ev****

Kazan Federal University
ul. Kremlyovskaya 18, Kazan, 420008 Russia

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Abstract—We describe possibilities of realizing V. K. Ivanov's method for solving inverse problems of logarithmic potential and that of A. V. Tsirul'skii's method for solving direct problems of logarithmic potential. We solve two direct problems for logarithmic potential and for simple layer potential in the case of polygonal contour.

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1. In problems of logarithmic potential we consider a simply connected plane domain D, filled with a homogeneous gravitational mass with a constant density μ, and representation of the gradient of potential $u(z)$ in a neighborhood of ∞. The representation of the gradient in the form of single-valued analytic function with zero of the first order at ∞ can be replaced by representation of the potential itself with logarithmic singularity at ∞.

In the direct problems the domain D is known, and we need to find the potential gradient outside and inside D. In the inverse problems, the potential gradient is known at a neighborhood of ∞, and we have to determine the domain D. Following Ivanov [1], we will continue to find classes of domains, described by a finite number of parameters, which characterize conformal mappings of the unit disk $E = \{|\zeta| < 1\}$ onto D.

2. Solving the inverse problem of logarithmic potential depending on a finite number of parameters is connected with Ivanov's equation ([1], [2], Chap. 4, § 3)

$$\mu z^*(t) = -\frac{1}{2\pi i}\int_\Gamma \frac{u[z(\tau)]d\tau}{\tau - t} \tag{1}$$

where $\Gamma = \{\tau : |\tau| = 1\}$, $z = z(\tau)$ is the equation of the boundary of the desired domain, $z^*(t) = \overline{z(1/\overline{t})}$, $u(z) = c_0/z + \sum\limits_{k=2}^{\infty} c_k/z^k$ is the given gradient at a neighborhood of ∞, which can be extended up to singularities of $u(z)$, μ is the constant density of the homogeneous mass, filling the domain D with boundary $\partial D = L = z(\Gamma)$.

When we specify

$$u(z) = -\frac{2}{\pi}\frac{\partial v(z)}{\partial z},\ \ v(z) = \iint_D \mu \ln\frac{1}{\tau - z} ds_\tau,$$

$$u(z) = -\frac{2}{\pi}\iint_D \frac{\mu}{\tau - z} ds_\tau \tag{2}$$

as a function with a finite number of singularities, *two cases* are possible (ds_τ is the area element).

*E-mail: Nail.Abubakirov@kpfu.ru.
**E-mail: Leonid.Aksentev@kpfu.ru.