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Cylindrical Tank Filled With a Liquid in a Three-Dimensional Temperature Field

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Abstract—In the cylindrical coordinate system, we construct an exact solution of the three-dimensional thermoelasticity problem for a tank filled with a liquid. After determining the temperature field from the heat conduction equation, we solve the equations of the asymmetrical problem of the theory of elasticity. In doing so, the system of resolving equations is reduced to four separate equations with respect to the displacements of the construction. Several exact solutions of boundary-value problems are found. The results are presented in the form of rather simple formulas.

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In the cylindrical coordinate system, the most compact form of the equations of the three-dimensional elasticity theory, which are written in terms of the displacements and stresses, is given in the monographs [1, 2]. Analogous equations with allowance for the temperature terms, which were constructed using the Duhamel–Neumann relations, are presented in the monograph [3]. All these equations have a common disadvantage, namely, it is not possible to obtain their integrable combinations, which leads to insurmountable difficulties in constructing exact solutions. In the monograph [4], this problem was solved for equations that do not contain temperature terms by means of introducing into the system an additional equation with respect to the volume deformation. An exact solution to this equation was already published in the work [1], but the authors of the work [4] succeeded in using the equation to solve the entire system of resolving relations.

The aim of this work is to construct the system of equations of the elasticity theory, which are most convenient for integration, and to find the solution of these equations.

As initial equations, we take those for a circular cylinder [3]. Without taking into account the mass forces, they can be written with respect to the displacements as follows:

$$\begin{gathered}
\Delta w + \frac{R}{(1-2\upsilon)}\left[\frac{1}{\varepsilon}\frac{\partial\theta}{\partial\gamma} - 2\,(1+\upsilon)\,\alpha_T\frac{1}{\varepsilon}\frac{\partial T}{\partial\gamma}\right] = 0,\\
\left(\Delta - \frac{1}{\alpha^2}\right)u + \frac{R}{(1-2\upsilon)}\left[\frac{\partial\theta}{\partial\alpha} - 2\,(1+\upsilon)\,\alpha_T\frac{\partial T}{\partial\alpha}\right] - \frac{2}{\alpha^2}\frac{\partial v}{\partial\beta} = 0,\\
\left(\Delta - \frac{1}{\alpha^2}\right)v + \frac{R}{(1-2\upsilon)}\left[\frac{1}{\alpha}\frac{\partial\theta}{\partial\beta} - 2\,(1+\upsilon)\,\alpha_T\frac{1}{\alpha}\frac{\partial T}{\partial\beta}\right] + \frac{2}{\alpha^2}\frac{\partial u}{\partial\beta} = 0,\\
\theta = \frac{1}{R}\left[\frac{1}{\alpha}\frac{\partial\,(\alpha\,u)}{\partial\alpha} + \frac{1}{\alpha}\frac{\partial v}{\partial\beta} + \frac{1}{\varepsilon}\frac{\partial w}{\partial\gamma}\right].
\end{gathered} \tag{1}$$

Here α, β, and γ are the dimensionless cylindrical coordinates (α is scaled by the outer radius R of the cylinder, γ is scaled by the height H of the cylinder, β is the angular coordinate along the guide); r is the radius of the inner lateral surface; u, v, and w are the displacements along the coordinate

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