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C^*-Algebras Generated by Mappings. Criterion of Irreducibility

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Abstract—We study the operator algebra associated with a self-mapping φ on a countable set X which can be represented as a directed graph. The algebra is generated by the family of partial isometries acting on the corresponding $l^2(X)$. We study the structure of involutive semigroup multiplicatively generated by the family of partial isometries. We formulate the criterion when the algebra is irreducible on the Hilbert space. We consider the concrete examples of operator algebras. In particular, we give the examples of nonisomorphic C^*-algebras, which are the extensions by compact operators of the algebra of continuous functions on the unit circle.

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INTRODUCTION

In this paper we study an operator algebra associated with a self-mapping of a countable set such that the preimage of each point is finite.

An algebraic approach to the theory of abstract dynamics systems was proposed by von Neumann [1]. The main idea of the algebraic theory of dynamical systems is in the construction of a C^*-algebra (or W^*-, as of von Neumann) that reflects the structure of the given dynamics system. For reversible systems, the dynamics is given by a group of automorphisms on measurable spaces. This theory is well investigated and enjoys wide acceptance [2–6].

The irreversible dynamics systems were investigated, e.g., in [7–11]. In [7] they consider a transform T preserving the measure (in [8], type of measure) of the Lebesgue space. In [9, 10] they consider a surjective mapping $T : X \longrightarrow X$, where X is a compactum.

In [12, 13], we proposed the construction of a C^*-algebra $C^*_\varphi(X)$ generated by a self-mapping $\varphi : X \longrightarrow X$ of a set. Unlike in the cited articles, we assume the set X to be countable and have no additional structure, but we do not assume the mapping φ to be reversible, we only assume it to satisfy the condition $\operatorname{card}\varphi^{-1}(x) < \infty$ for any $x \in X$. A family $\mathcal{U}$ of partial isometries acting on $l^2(X)$, finite or countable, is connected with a pair (X, φ). These are the isometries that generate $C^*_\varphi(X)$ and, in addition, satisfy the relations: The sum of the initial and final projections results into noncommuting projections defined by the mapping φ. Therefore, in some sense, $C^*_\varphi(X)$ can be classified as an algebra generated by partial isometries with some algebraic relations. The classical examples of

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