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A Stokes–Brinkman model of the fluid flow in a periodic cell with a porous body using the boundary element method

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ABSTRACT

The problem of viscous incompressible flow in a periodic cell with a porous body is solved. The Stokes flow model is adopted to describe the flow outside the body and the Brinkman equation is applied to find the filtration velocity field inside the porous domain. The conditions on the boundary between the free fluid and the porous medium for the porous body of arbitrary shape are obtained. The boundary value problem for the joint solution of the biharmonic and Brinkman equations for the stream functions outside and inside the porous body are then solved using a boundary element method. Good agreement of the numerical and analytical models for the Kuwabara circular cell model is shown for the fluid flow through a porous circular cylinder. The fluid flow past a circular, square, triangular cylinders and a circular body of uneven surface (an idealized model of a viral capsid) in a rectangular periodic cell are calculated. Comparison of the results obtained with the numerical solution from a CFD ANSYS/FLUENT model shows good accuracy of the developed mathematical model.

1. Introduction

The solution of the problem of fluid flow through porous bodies is used to describe many hydrodynamical processes in environmental and medical science. For example, such flows are found in aerosol filters and respirators and for particle shaped viruses in biofluids. In the case of aerosol filters porous bodies can be used as elements to increase the efficiency of the deposition of aerosol particles [1,2]. In order to calculate the two-phase flows of dusty air for such filters it is important to develop efficient mathematical models of fluid flow past porous bodies in periodic cells [3].

The fluid flow through a circular porous cylinder assuming potential flow of an incompressible fluid was modelled in work [4].

One of the problems with modelling fluid flow in domains containing porous medium is connected with the formulation of the boundary conditions on the interface between the free space and porous medium. The choice of the conditions depends on the mathematical model adopted. Various boundary conditions are studied in the works of Beavers and Joseph [5], Saffman [6], Neale and Nader [7], Haber and Mauri [8], Vafai and Thiyagaraja [9], Sahraoui and Kaviany [10], Ochoa-Tapia and Whitaker [11,12].

The analytical solution of the problem of the fluid flow past an isolated porous cylinder and a system of porous cylinders using a cell model was firstly obtained by Stechkina [13]. The cell model used was based on the widely adopted Kuwabara cell model [14] and included the Stokes flow model [15] outside and the Brinkman equation [16] inside the porous cylinder. The cell model with Kuwabara boundary conditions was also used by Deo et al. [17] and Kirsh [1] to determine the velocity field of the flow over and through a porous cylinder in the case of small Reynolds number flow using the analytical solution and the collocation method.

A review of analytical investigations of fluid flow past porous cylinders and spheres is given in the works of Deo et al. [18] and Vasin and Filippov [19]. Generally, the cell model is an approximate model of fluid flow and its accuracy depends on the porosity of the porous medium. To obtain an accurate fluid flow velocity field numerical models using the real array geometry should be adopted.

Viscous flow models for flows through porous bodies usually adopt the combination of the Stokes model in the free space and the Darcy or Brinkman model in the porous medium. Such models were used to study the viscous flow through isolated porous cylinders and spheres by Masliyah and Polikar [20], Nandakumar and Masliyah [21], Noymer et al [22], Vanni [23], Vainshtein et al [24,25].

The Navier–Stokes equations with Darcy and Forchmeier terms were solved numerically to simulate the fluid flow past a porous cylinder in Beckermann and Viskanta [26], Vafai and Kim [27], Basu and Khalili [28], Bhattacharyya et al [29].

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