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Riesz-Fejér Inequalities for Harmonic Functions

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Abstract
In this article, we prove the Riesz - Fejér inequality for complex-valued harmonic functions in the harmonic Hardy space $\mathbf{h}^p$ for all $p > 1$. The result is sharp for $p \in (1, 2]$. Moreover, we prove two variant forms of Riesz-Fejér inequality for harmonic functions, for the special case $p = 2$.

Keywords Riesz - Fejér type inequalities · Integral means · Harmonic hardy spaces

Mathematics Subject Classification (2010) Primary: 31A05, 30H10

1 Introduction and Main Results

Let $\mathbb{D}$ be the open unit disk in the complex plane, i.e. $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$ and let $\mathcal{A}$ denote the class of all analytic functions f defined on $\mathbb{D}$. For $f \in \mathcal{A}$, the integral means $M_p(r, f)$ is defined as

$$M_p(r, f) = \left(\frac{1}{2\pi} \int_0^{2\pi} |f(re^{i\theta})|^p d\theta \right)^{1/p}, \quad 0 < p < \infty.$$

The classical Hardy space H^p, $0 < p < \infty$, consists of all analytic functions $f : \mathbb{D} \to \mathbb{C}$ such that $M_p(r, f)$ remains bounded as $r \to 1^-$. The study of H^p spaces attracted the attention of many mathematicians, as it deals with the important problems in function theory such as the existence of the radial limit in almost all directions, growth of the absolute value

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