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Limit vector variational inequality problems via scalarization

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Abstract We solve a general vector variational inequality problem in a finite—dimensional setting, where only approximation sequences are known instead of exact values of the cost mapping and feasible set. We establish a new equivalence property, which enables us to replace each vector variational inequality with a scalar set-valued variational inequality. Then, we approximate the scalar set-valued variational inequality with a sequence of penalized problems, and we study the convergence of their solutions to solutions of the original one.

Keywords Vector variational inequality · Non-stationarity · Set-valued mappings · Approximation sequence · Penalty method · Coercivity conditions

1 Introduction

Let D be a nonempty convex set in the real n-dimensional space $\mathbb{R}^n$, and let $G : D \to \mathbb{R}^n$ be a mapping. Then one can define the *variational inequality problem* (VI, for short), which is to find an element $x^* \in D$ such that

$$\langle G(x^*), y - x^* \rangle \geq 0 \quad \forall y \in D. \tag{VI}$$

VIs give a suitable common format for various applied problems and are closely related with other general problems in nonlinear analysis, such as fixed point, optimization, complementarity, and equilibrium problems; see, e.g., [4–9] and the references therein. Moreover,

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