



# Higher- $n$ triangular dilatonic black holes

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## ABSTRACT

Dilaton gravity with the form fields is known to possess dyon solutions with two horizons for the discrete “triangular” values of the dilaton coupling constant  $a = \sqrt{n(n+1)}/2$ . This sequence first obtained numerically and then explained analytically as consequence of the regularity of the dilaton, should have some higher-dimensional and/or group theoretical origin. Meanwhile, this origin was explained earlier only for  $n = 1, 2$  in which cases the solutions were known analytically. We extend this explanation to  $n = 3, 5$  presenting analytical triangular solutions for the theory with different dilaton couplings  $a, b$  in electric and magnetic sectors in which case the quantization condition reads  $ab = n(n+1)/2$ . The solutions are derived via the Toda chains for  $B_2$  and  $G_2$  Lie algebras. They are found in the closed form in general  $D$  space–time dimensions. Solutions satisfy the entropy product rules indicating on the microscopic origin of their entropy and have negative binding energy in the extremal case.

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## 1. Introduction

Einstein–Maxwell–dilaton (EMD) theory in four dimensions may originate from different supersymmetric higher-dimensional theories with various values of the dilaton coupling constant  $a$  in the Maxwell term  $e^{-2a\phi} F^2$  in the Lagrangian. For  $a = 0$  the dilaton decouples and the theory reduces to Einstein–Maxwell (EM) system, which is the bosonic part of  $N = 2, D = 4$  supergravity. The value  $a = 1$  corresponds to  $N = 4, D = 4$  supergravity [1,2]. The value  $a = \sqrt{3}$ , corresponds to dimensionally reduced  $D = 5$  gravity [3,4], which have different supersymmetric extensions. In all these cases analytical solutions are known for static black holes possessing both electric and magnetic charges (dyons) with two horizons between which the dilaton exhibits  $n$  oscillations. Such solutions have non-singular extremal limits with  $\text{AdS}_2 \times S^2$  horizons contrary the one-charged dilatonic black hole which have singular horizons in the extremal limit [1,2,5] for generic coupling constant.

As it was first noticed by Poletti, Twamley and Wiltshire [6], the values  $a = 1, \sqrt{3}$  are just the two lowest members  $n = 1, 2$  of the “triangular” sequence of dilaton couplings

$$a_n = \sqrt{n(n+1)}/2, \quad (1)$$

for which dyons (known for higher  $n$  only numerically) exhibit similar behavior of the dilaton. Analytically this triangle quantization rule was rederived in [7] as condition of regularity of the dilaton in the case of coinciding horizons, i.e. for extremal dyons. The same rule was shown to arise from the linearized dilaton equation on the Reissner–Nordström dyonic background [8,9] as condition of existence of dilaton bound states. However, exact higher- $n$  dyonic solutions were not known analytically. Here we give solutions for  $n = 3, 5$  in the theory where the dilaton couples to electric and magnetic sectors with different coupling constants  $a, b$ , in which case the quantization rule generalizes to  $ab = n(n+1)/2$ . We also give generalization to arbitrary dimensions. These new solutions are derived via Toda chains for  $B_2$  and  $G_2$  algebras. The solutions satisfy the entropy product rule: the product of the entropies of the internal and external horizons are functions of their charges only [10,11] (see also [8]). This property is considered to be an indication on possibility of statistical interpretation of the entropy. Another interesting feature is that the extremal dyons have negative binding energy like the  $SL(n, R)$  multi-scalar Toda black holes [12].

Previous work on Toda dilatonic black holes includes among other [13–16]. Our results have some overlap with the recent preprints [9,17].

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