

# Lower bounds and hierarchies for quantum memoryless communication protocols and quantum ordered binary decision diagrams with repeated test

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## Abstract

© 2018, Springer International Publishing AG. We explore multi-round quantum memoryless communication protocols. These are restricted version of multi-round quantum communication protocols. The “memoryless” term means that players forget history from previous rounds, and their behavior is obtained only by input and message from the opposite player. The model is interesting because this allows us to get lower bounds for models like automata, Ordered Binary Decision Diagrams and streaming algorithms. At the same time, we can prove stronger results with this restriction. We present a lower bound for quantum memoryless protocols. Additionally, we show a lower bound for Disjointness function for this model. As an application of communication complexity results, we consider Quantum Ordered Read-k-times Branching Programs (k-QOBDD). Our communication complexity result allows us to get lower bound for k-QOBDD and to prove hierarchies for sublinear width bounded error k-QOBDDs, where  $k=o(n)$ . Furthermore, we prove a hierarchy for polynomial size bounded error k-QOBDDs for constant k. This result differs from the situation with an unbounded error where it is known that an increase of k does not give any advantage.

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## Keywords

Binary decision diagrams, Branching programs, Communication complexity, Computational complexity, Hierarchy, OBDD, Quantum computation, Quantum models

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