Theoretical and Mathematical Physics, **194**(3): 377–383 (2018)

ZONE STRUCTURE OF THE RENORMALIZATION GROUP FLOW IN A FERMIONIC HIERARCHICAL MODEL

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The Gaussian part of the Hamiltonian of the four-component fermion model on a hierarchical lattice is invariant under the block-spin transformation of the renormalization group with a given degree of normalization (the renormalization group parameter). We describe the renormalization group transformation in the space of coefficients defining the Grassmann-valued density of a free measure as a homogeneous quadratic map. We interpret this space as a two-dimensional projective space and visualize it as a disk. If the renormalization group parameter is greater than the lattice dimension, then the unique attractive fixed point of the renormalization group is given by the density of the Grassmann delta function. This fixed point has two different (left and right) invariant neighborhoods. Based on this, we classify the points of the projective plane according to how they tend to the attracting point (on the left or right) under iterations of the map. We discuss the zone structure of the obtained regions and show that the global flow of the renormalization group is described simply in terms of this zone structure.

Keywords: renormalization group, fermion model, projective space, zone structure

DOI: 10.1134/S0040577918030054

1. Introduction

In [1]–[6], we studied the transformation properties of the renormalization group (RG) in the framework of the fermionic hierarchical model. This model is a fermionic (anticommutative) analogue of the hierarchical boson Dyson φ^4 model. The transformation of the block Wilson RG in the framework of the bosonic hierarchical model reduces to a nonlinear integral operator in an infinite-dimensional space of free measure densities. The properties of this transformation in the neighborhood of a Gaussian fixed point were studied in the works of Blecher and Sinai and many other authors [7]–[9]. In contrast to the bosonic case, the RG transformation in the hierarchical fermion model can be calculated exactly and represented as a birational map in the two-dimensional space of coupling constants of the model. This allows describing all the fixed points and other dynamical properties and symmetries of the RG map explicitly. We also note that the fermionic hierarchical model has a natural continuous p-adic version if the size of the elementary block of the hierarchical lattice is equal to a prime p (see [6], [10], [11]). In this case, problems in quantum field theory such as ultraviolet divergences and the renormalization procedure have a natural interpretation in terms of the classical theory of dynamical systems.

In [4], we described some of the global dynamical properties of the RG flow in the plane of coupling constants of the model (r, g). Analyzing the RG flow with a computer, we found that almost all points of the upper half-plane $\{(r, g)\colon g > 0\}$, which is invariant under the RG map, under successive iterations of the RG map go to infinity "from the left" ($r \to -\infty$, $g \to \infty$) or "from the right" ($r \to \infty$, $g \to \infty$). We

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Translated from Teoreticheskaya i Matematicheskaya Fizika, Vol. 194, No. 3, pp. 436–444, March, 2018. Original article submitted February 2, 2017.

0040-5779/18/1943-0377 © 2018 Pleiades Publishing, Ltd.