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CONTRIBUTION TO THE GENERAL LINEAR CONJUGATION PROBLEM FOR A PIECEWISE ANALYTIC VECTOR

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Abstract: Establishing an analogy between the theories of Riemann–Hilbert vector problem and linear ODEs, for the n-dimensional homogeneous linear conjugation problem on a simple smooth closed contour Γ partitioning the complex plane into two domains D^+ and D^- we show that if we know $n-1$ particular solutions such that the determinant of the size $n-1$ matrix of their components omitting those with index k is nonvanishing on $D^+ \cup \Gamma$ and the determinant of the matrix of their components omitting those with index j is nonvanishing on $\Gamma \cup D^- \setminus \{\infty\}$, where $k, j = \overline{1,n}$, then the canonical system of solutions to the linear conjugation problem can be constructed in closed form.

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Take a simple smooth closed contour Γ partitioning the complex plane into two domains D^+ and D^-, with $0 \in D^+$ and $\infty \in D^-$, and consider a size n matrix function H-continuous on Γ

$$G(t) = \begin{pmatrix} g_{11}(t) & g_{12}(t) & \dots & g_{1n}(t) \\ g_{21}(t) & g_{22}(t) & \dots & g_{2n}(t) \\ \dots & \dots & \dots & \dots \\ g_{n1}(t) & g_{n2}(t) & \dots & g_{nn}(t) \end{pmatrix}, \quad \Delta(t) = \det G(t) \neq 0,\ t \in \Gamma. \tag{1}$$

The homogeneous linear conjugation problem for an n-dimensional vector, or the vector Riemann–Hilbert problem, consists in finding a piecewise analytic vector function $\mathbf{w}(z) = (w^1(z), w^2(z), \dots, w^n(z))$ of the specified order at infinity with the limit values $\mathbf{w}^{\pm}(t)$ H-continuous on Γ and subject to the condition

$$\mathbf{w}^+(t) = G(t)\mathbf{w}^-(t)$$

or, in scalar form, to the conditions

$$\begin{aligned} w^{1+}(t) &= g_{11}(t)w^{1-}(t) + g_{12}(t)w^{2-}(t) + \dots + g_{1n}(t)w^{n-}(t), \\ w^{2+}(t) &= g_{21}(t)w^{1-}(t) + g_{22}(t)w^{2-}(t) + \dots + g_{2n}(t)w^{n-}(t), \\ &\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ w^{n+}(t) &= g_{n1}(t)w^{1-}(t) + g_{n2}(t)w^{2-}(t) + \dots + g_{nn}(t)w^{n-}(t). \end{aligned} \tag{2}$$

The qualitative theory of problem (2) in the classes of Hölder functions of arbitrary dimension is presented in [1], and for wider classes of matrix functions, in [2]. However, there are few examples of matrix functions for which the solution to the problem can be expressed in closed form via Cauchy-type integrals and solutions to certain linear algebraic systems. One of these examples is the linear conjugation problem for meromorphic matrix functions [3]. A constructive algorithm for solving this problem is also proposed in [2]. An algorithm is presented in [4] for efficiently constructing a canonical system of solutions whenever n solutions to the linear conjugation problem are available such that the determinant consisting of their components has finitely many zeros in the corresponding domain. The author demonstrated in [5] the possibility of constructing in closed form a canonical system of solutions to the linear conjugation problem for a three-dimensional vector in the presence of just two particular solutions of dimension is less by 1 than the dimension of the problem. A constructive algorithm is presented in [6] for implementing a right Wiener–Hopf factorization on the real axis from the available $n-1$ solutions to the n-dimensional

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