Journal of Fluid Mechanics 2018 vol.856, pages 673-708

Almost limiting configurations of steady interfacial overhanging gravity waves

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Abstract

© 2018 Cambridge University Press. We study progressive gravity waves at the interface between two unbounded fluids of different densities. The main concern is to find almost limiting configurations for the so-called overhanging waves. The latter were first computed by Meiron & Saffman (J. Fluid Mech., vol. 129, 1983, pp. 213-218). By means of the Hopf lemma, we rigorously prove that, if is the angle between the tangent line to the interfacial curve and the horizontal direction, then < π θ < π x03C0}. Analysing the results of computations, we introduce two new concepts: an inner crest, and an inner solution near the inner crest. These concepts allow us to make a well-grounded prediction for the shapes of limiting interfacial configurations and confirm Saffman & Yuen's (J. Fluid Mech., vol. 123, 1982, pp. 459-476) conjecture that the waves are geometrically limited.

http://dx.doi.org/10.1017/jfm.2018.721

Keywords

computational methods, interfacial flows (free surface), internal waves

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