

Initial Value Problem for B -Hyperbolic Equation with Integral Condition of the Second Kind

K. B. Sabitov^{a,b*} and N. V. Zaitseva^{c**}

^a*Bashkortostan Institute for Strategic Research, Sterlitamak, Bashkortostan, 453103 Russia*

^b*Sterlitamak Branch of Bashkir State University, Sterlitamak, Bashkortostan, 453103 Russia*

^c*Kazan (Volga) Federal University, Kazan, Tatarstan, 420008 Russia*

*e-mail: *sabitov_fmfm@mail.ru, **n.v.zaitseva@yandex.ru*

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Abstract—For the hyperbolic equation with Bessel operator, we study the initial boundary-value problem with integral nonlocal condition of the second kind in a rectangular domain. The integral identity method is used to prove the uniqueness of the solution to the posed problem. The solution is constructed as a Fourier–Bessel series. To justify the existence of the solution to the nonlocal problem, we obtain sufficient conditions to be imposed on the initial conditions to ensure the convergence of the constructed series in the class of regular solutions.

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1. INTRODUCTION

We consider the hyperbolic equation with Bessel operator

$$\square_B u(x, t) \equiv u_{tt} - x^{-k} \frac{\partial}{\partial x} (x^k u_x) = 0, \quad (1)$$

where $x > 0$ and k is a given real number, $k > -1$, $k \neq 0$. Equation (1), which, after Kipriyanov [1, p. 7], will be called a B -hyperbolic equation, arises, for example, when switching from Cartesian to cylindrical coordinates in the wave equation while considering radial gas vibrations in a stationary infinite cylindrical pipe or when switching to spherical coordinates while considering small vibrations of a gas near its equilibrium position inside an impermeable spherical shell [2, pp. 185, 191]. Pul'kin [3] studied the Cauchy and Cauchy–Goursat problems for Eq. (1) with $k \geq 1$. The Tricomi problem for a mixed-type equation with a hyperbolic part that coincides with Eq. (1) was considered in [4, 5].

Let $D = \{(x, t) : 0 < x < l, 0 < t < T\}$ be a rectangular domain in the Oxt coordinate plane, where $l, T > 0$ are given real numbers. In the present paper, for Eq. (1) in the domain D , we study the following initial value problems with a nonlocal boundary condition of the second kind for $k \geq 1$ and $-1 < k < 1$, $k \neq 0$.

Problem 1. Let $k \geq 1$. To determine such a solution $u(x, t)$ of the equation

$$\square_B u(x, t) \equiv 0, \quad (x, t) \in D, \quad (2)$$

that

$$u(x, t) \in C^1(\overline{D}) \cap C^2(D) \quad (3)$$

and it satisfies the initial conditions

$$u(x, 0) = \varphi(x), \quad u_t(x, 0) = \psi(x), \quad 0 \leq x \leq l, \quad (4)$$