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TURING DEGREES IN REFINEMENTS OF THE ARITHMETICAL HIERARCHY

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We investigate the problem of characterizing proper levels of the fine hierarchy (up to Turing equivalence). It is known that the fine hierarchy exhausts arithmetical sets and contains as a small fragment finite levels of Ershov hierarchies (relativized to $\varnothing^n$, $n < \omega$), which are known to be proper. Our main result is finding a least new (i.e., distinct from the levels of the relativized Ershov hierarchies) proper level. We also show that not all new levels are proper.

INTRODUCTION

Investigation of Turing degrees of Δ^0_2-sets is a central topic in local degree theory. The structure of Δ^0_2-degrees turned out to be quite rich and complex, and extending the local degree theory to higher levels of the arithmetical hierarchy leads to additional complications. In this paper, we consider some natural questions about such extensions related to one direction in local degree theory, namely to investigating degrees in the Ershov hierarchy.

This line of research was initiated by B. Cooper [1] who constructed a difference of computably enumerable (c.e.) sets which is not (Turing) equivalent to a c.e. set. Since the c.e. sets form the first level Σ^{-1}_1 of Ershov's hierarchy $\{\Sigma^{-1}_k\}_{k<\omega}$ [2] and the differences of c.e. sets form its second level, we can ask about the possibility to extend Cooper's result to all levels. This was independently

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