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Two classes of τ-measurable operators affiliated with a von Neumann algebra

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Abstract

Let M be a von Neumann algebra of operators on a Hilbert space H, τ be a faithful normal semifinite trace on M. We define two (closed in the topology of convergence in measure τ) classes P1 and P2 of τ-measurable operators and investigate their properties. The class P2 contains P1. If a τ-measurable operator T is hyponormal, then T lies in P1; if an operator T lies in Pk, then UTU* belongs to Pk for all isometries U from M and k = 1, 2; if an operator T from P1 admits the bounded inverse T−1, then T−1 lies in P1. We establish some new inequalities for rearrangements of operators from P1. If a τ-measurable operator T is hyponormal and Tn is τ-compact for some natural number n, then T is both normal and τ-compact. If M = B(H) and τ = tr, then the class P1 coincides with the set of all paranormal operators on H.

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Keywords

Hilbert space, hyponormal operator, integrable operator, normal trace, paranormal operator, projection, quasinormal operator, rearrangement, topology of convergence in measure, von Neumann algebra, τ-compact operator, τ-measurable operator

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