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Rainfall induced groundwater mound in wedge-shaped promontories: The Strack–Chernyshov model revisited

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ABSTRACT

An analytical solution to the Poisson equation governing Strack's discharge potential (squared thickness of a saturated zone in an unconfined aquifer) is obtained in a wedge-shaped domain with given head boundary conditions on the wedge sides (specified water level in an open water body around a porous promontory). The discharge vector components, maximum elevation of the water table in promontory vertical cross-sections, quantity of groundwater seeping through segments of the wedge sides, the volume of fresh groundwater in the mound are found. For acute angles, the solution to the problem is non-unique and specification of the behaviour at infinity is needed. A "basic" solution is distinguished, which minimizes the water table height above a horizontal bedrock. MODFLOW simulations are carried out in a finite triangular island and compare solutions with a constant-head, no-flow and "basic" boundary condition on one side of the triangle. Far from the tip of an infinite-size promontory one has to be cautious with truncation of the simulated flow domains and imposing corresponding boundary conditions. For a right and obtuse wedge angles, there are no positive solutions for the case of constant accretion on the water table. In a particular case of a confined rigid wedge-shaped aquifer and incompressible fluid, from an explicit solution to the Laplace equation for the hydraulic head with arbitrary time-space varying boundary conditions along the promontory rays, essentially 2-D transient Darcian flows within the wedge are computed. They illustrate that surface water waves on the promontory boundaries can generate strong Darcian waves inside the porous wedge. Evaporation from the water table and sea-water intruded interface (rather than a horizontal bed) are straightforward generalizations for the Poissonian Strack potential.

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"See one promontory, one mountain, one sea, one river and see all."

Socrates

1. Introduction

Groundwater mounds or troughs are naturally recharged/evaporate across the water table, with infiltrated/exfiltrated water moving – quasi-vertically – from/to the vadose zone (Fig. 1) to/from a shallow aquifer. A quasi-horizontal motion within this unconfined aquifer is towards/from surface water bodies, atmosphere or draining topographic depressions (e.g. sea/lake shore, network of streams, springs, combes, interstices, gullies, among others). The juxtaposition of the vadose zone and aquifer flows is essentially 2-,3-D (see e.g. Broadbridge et al., 1988; Ameli et al., 2013; Toth, 2009; Tritscher et al., 2001).

In this paper, we consider a steady Darcian flow in a homogeneous and isotropic aquifer with a horizontal bed and assume the Dupuit–Forchheimer approximation, i.e. eliminate the vertical coordinate as an independent variable. Then in a physical domain D (planar projection of the infiltrated-evaporated area) we determine the mound elevation, $h(x,y)$, above an aquifer bed (horizontal plane) and the discharge vector $\vec{Q}(x,y)$(see, e.g., Haitjema, 1995; PK-77, Strack, 1989; Youngs, 1992). The corresponding boundary value problem (BVP) in a planar domain D, bounded by a draining contour Γ, is formulated in notations of Strack's potential (1989, Ch.2, Section 9):

$$\begin{aligned} &\vec{Q} = -\nabla\Phi, \\ &\Delta\Phi(x,y) = -N \\ &\Phi_\Gamma = \Phi_0 = k\frac{h_0{}^2}{2} = const \end{aligned} \tag{1}$$

Abbreviations: BVP, Boundary value problem; PDE, Partial differential equation; PK-77, Polubarinova-Kochina, 1977; SWI, Sea water intrusion.

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