

## On hermitian operators $X$ and $Y$ meeting the condition $-Y \leq X \leq Y$

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### Abstract

We obtain a description of all pairs of Hermitian operators  $X$  and  $Y$ , which satisfy the condition  $-Y \leq X \leq Y$ . We give the examples of such operator pairs. Each of the presented examples leads us to the new weak majorization for the Hermitian operator pair. It is shown that this inequality does not necessarily imply the inequality  $\|X\| \leq \|ZY Z^*\|$  for any operator  $Z$ ,  $\|Z\| \leq 1$ . We prove that invertibility of  $Y$  follows from invertibility of operators  $X$  and  $A^* A$  for Hermitian operators  $X$  and  $Y$ ,  $Y \geq 0$  and an arbitrary operator  $A$  such that  $-AY A^* \leq X \leq AY A^*$ . We discuss one analog of triangle inequality found by the author in one earlier paper for pairs of Hermitian operators. © 2013 Pleiades Publishing, Ltd.

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### Keywords

Hilbert space, linear bounded operator, matrix, operator inequality, projection, unitary operator, von Neumann algebra, weak majorization