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Self-force on a scalar point charge in the long throat

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ABSTRACT

An analytic method is presented which allows for the computation of the self-force for a static particle with a scalar charge in the region of an ultrastatic spacetime which one can call the long throat. The method is based on the approximate WKB solution of a radial mode equation for a scalar field. This field is assumed to be massless, with a coupling ξ to the scalar curvature is satisfying the condition $\xi > 1/8$.

1. Introduction

The study of a self-force has a long history. The original investigations focused on the self-acceleration of an electrically-charged point particle in flat spacetime [1]. Later DeWitt, Brehme, and Hobbs [2] studied the influence of the self-force on a charge in a curved spacetime. In contrast to the case of a flat spacetime this force can be nonzero even for a static charge in a curved background. A number of static configurations has been analyzed, including the self-action in the spacetimes of a Schwarzschild black hole [3,4], of a Kerr black hole [5], of a Kerr–Newman black hole [4] and in a spherically symmetric Brans–Dicke field [6]. The analytic approximation of self-force has been obtained for a scalar charge at rest in an axisymmetric spacetime [7]. The self-force can be nonzero for a static particle in flat spacetimes of the topological defects [8]. In curved spacetimes with nontrivial topological structure the investigations of this type have the additional interesting features [9,10].

The effect of self-action is associated with nonlocal structure of the massless field, the source of which is the charged particle. For example, the self-force on a scalar charge is [11]

$$f_\mu = q^2\left[\frac{1}{3}\left(\dot a_\mu - a^2 u_\mu\right) + \frac{1}{6}\left(R^\nu_\mu u_\nu + R_{\nu\gamma}u^\nu u^\gamma u_\mu\right) + \frac{1}{12}(6\xi - 1)R u_\mu + \lim_{\epsilon\to 0}\int_{-\infty}^{\tau-\epsilon} \nabla_\mu G_{ret}\left(x, x'\right) d\tau'\right] \qquad (1)$$

where u_μ is the 4-velocity of the particle, a_μ is the 4-acceleration, $\dot a_\mu = \partial a_\mu/\partial\tau$ is the derivative of the 4-acceleration with respect to proper time τ of a charged particle, $G_{ret}(x, x')$ is the retarded scalar Green's function and ξ is the coupling to the background scalar curvature.

Are there the situations in which the effect of self-action is determined by the local geometry of the curved spacetime? As it is demonstrated below such a situation for the static scalar charge takes place, for example, in the throat of the wormhole if the length of this throat is much more than the radius of throat. As the examples of such wormholes one can consider the spacetimes with metric

$$ds^2 = -dt^2 + d\rho^2 + \left(r_0 + \rho \tanh\frac{\rho}{\rho_0}\right)^2 \left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \qquad (2)$$

or

$$ds^2 = -dt^2 + d\rho^2 + \left(r_0 + \rho \coth\frac{\rho}{\rho_0} - \rho_0\right)^2 \left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \qquad (3)$$

where r_0, ρ_0 are the constants (r_0 is a radius of the throat, ρ_0 is the parameter which describes the length of the throat) and

$$\frac{r_0}{\rho_0} \ll 1. \qquad (4)$$

The effect of self-action in the region $\rho \lesssim \rho_0$ does not depend on the geometry of a spacetime outside of this region and we shall

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