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Integration over non-rectifiable curves and Riemann boundary value problems

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ABSTRACT

In this paper we introduce an alternative way of defining the curvilinear Cauchy integral over non-rectifiable curves in the case of complex functions of one complex variable. Especially the jump behavior on the boundary is considered. As an application, solvability conditions of the Riemann boundary value problem are derived on very weak conditions to the boundary. Besides the complex case the consideration can be extended to the theory of Douglis algebra valued functions.

1. Introduction and preliminaries

The Riemann boundary value problem for analytic functions is well-known and has numerous applications in elasticity theory, hydro and aerodynamics, theory of orthogonal polynomials and so on.

Let Γ be a closed Jordan curve on the complex plane $\mathbb{C}$ bounding a bounded domain D^+, and $D^- = \overline{\mathbb{C}} \setminus \overline{D^+}$. We seek an analytic in $\overline{\mathbb{C}} \setminus \Gamma$ function $\Phi(z)$ such that $\Phi(\infty) = 0$, the boundary values $\lim_{D^+ \ni z \to t} \Phi(z) \equiv \Phi^+(t)$ and $\lim_{D^- \ni z \to t} \Phi(z) \equiv \Phi^-(t)$ exist for any $t \in \Gamma$, and

$$\Phi^+(t) = G(t)\Phi^-(t) + g(t), \quad t \in \Gamma, \tag{1}$$

where G and g are given functions. The simplest special case of (1) is the so-called jump problem:

$$\Phi^+(t) - \Phi^-(t) = g(t), \quad t \in \Gamma. \tag{2}$$

During the 19th century, this problem was the subject of investigations of Sokhotskii, Plemelj and others (see, for instance, [9,20]). In details, if the curve Γ is smooth and the jump $g(t)$ satisfies on Γ the Hölder condition with exponent $\nu \in (0, 1]$, then unique solution of this problem is the Cauchy integral

$$\Phi(z) = \frac{1}{2\pi i} \int_\Gamma \frac{g(\zeta)\, d\zeta}{\zeta - z}. \tag{3}$$

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