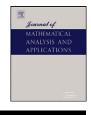


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Integration over non-rectifiable curves and Riemann boundary value problems

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ARTICLE INFO ABSTRACT Article history: In this paper we introduce an alternative way of defining the curvilinear Cauchy integral Received 10 January 2010 over non-rectifiable curves in the case of complex functions of one complex variable. Available online 26 February 2011 Especially the jump behavior on the boundary is considered. As an application, solvability Submitted by Steven G. Krantz conditions of the Riemann boundary value problem are derived on very weak conditions to the boundary. Besides the complex case the consideration can be extended to the theory Keywords: of Douglis algebra valued functions. Non-rectifiable curve © 2011 Elsevier Inc. All rights reserved. Fractional dimension Cauchy integral Douglis algebra Riemann boundary value problem

1. Introduction and preliminaries

The Riemann boundary value problem for analytic functions is well-known and has numerous applications in elasticity theory, hydro and aerodynamics, theory of orthogonal polynomials and so on.

Let Γ be a closed Jordan curve on the complex plane \mathbb{C} bounding a bounded domain D^+ , and $D^- = \overline{\mathbb{C}} \setminus \overline{D^+}$. We seek an analytic in $\overline{\mathbb{C}} \setminus \Gamma$ function $\Phi(z)$ such that $\Phi(\infty) = 0$, the boundary values $\lim_{D^+ \ni z \to t} \Phi(z) \equiv \Phi^+(t)$ and $\lim_{D^- \ni z \to t} \Phi(z) \equiv \Phi^-(t)$ exist for any $t \in \Gamma$, and

$$\Phi^+(t) = G(t)\Phi^-(t) + g(t), \quad t \in \Gamma,$$

where G and g are given functions. The simplest special case of (1) is the so-called jump problem:

$$\Phi^+(t) - \Phi^-(t) = g(t), \quad t \in \Gamma.$$

During the 19th century, this problem was the subject of investigations of Sokhotskii, Plemelj and others (see, for instance, [9,20]). In details, if the curve Γ is smooth and the jump g(t) satisfies on Γ the Hölder condition with exponent $\nu \in (0, 1]$, then unique solution of this problem is the Cauchy integral

$$\Phi(z) = \frac{1}{2\pi i} \int_{\Gamma} \frac{g(\zeta) \, d\zeta}{\zeta - z}.$$
(3)

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