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# Concerning the theory of τ-measurable operators affiliated to a semifinite von Neumann algebra

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## Abstract

 Let M be a von Neumann algebra of operators in a Hilbert space H, let τ be an exact normal semifinite trace on M, and let L1(M, τ) be the Banach space of τ-integrable operators. The following results are obtained. If X = X*, Y = Y* are τ-measurable operators and XY ∈ L1(M, τ), then YX ∈ L1(M, τ) and τ(XY) = τ(YX) ∈ R. In particular, if X, Y ∈ B(H)sa and XY ∈ G1, then YX ∈ G1 and tr(XY) = tr(YX) ∈ R. If X ∈ L1(M, τ), then (Formula Presented.). Let A be a τ-measurable operator. If the operator A is τ-compact and V ∈ M is a contraction, then it follows from V* AV = A that V A = AV. We have A = A2 if and only if A = |A*||A|. This representation is also new for bounded idempotents in H. If A = A2 ∈ L1(M, τ), then (Formula Presented.). If A = A2 and A (or A*) is semihyponormal, then A is normal, thus A is a projection. If A = A3 and A is hyponormal or cohyponormal, then A is normal, and thus A = A* ∈ M is the difference of two mutually orthogonal projections (A + A2)/2 and (A2 − A)/2. If A,A2 ∈ L1(M, τ) and A = A3, then τ(A) ∈ R.

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## Keywords

Banach space of τ-integrable operators, cohyponormal operator, Hilbert space, hyponormal operator, idempotent, semihyponormal operator, von Neumann algebra, τ-compact operator, τ-measurable operator