Journal of Geometry and Physics 9 (1992) 25–44
North-Holland

JOURNAL OF
GEOMETRY AND
PHYSICS

Quantization of Poisson pairs: the R-matrix approach

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Received 2 July 1990
(Revised 17 December 1990)

We suggest an approach to the quantization problem of two compatible Poisson brackets in the case when one of them is associated with a solution of the classical Yang–Baxter equation. We show that the quantization scheme (a Poisson bracket → an associate algebra, quantizing this bracket in the spirit of the Berezin–Lichnerovicz deformation quantization → its representation in a Hilbert space) has to be enlarged. We represent the deformation algebras, quantizing the "R-matrix" brackets, in a space with an S-symmetric pairing, where S is a solution of the corresponding quantum Yang–Baxter equation. An example of quantization of an "exotic" harmonic oscillator is discussed.

Keywords: *R-matrix, quantum Yang–Baxter equation, deformation quantization, cocycle*
MSC 1991: 81C99

An R-matrix is an invertible operator $R: V^{\otimes 2} \to V^{\otimes 2}$ satisfying the quantum Yang–Baxter equation (QYBE)

$$R^{12}R^{13}R^{23}=R^{23}R^{13}R^{12}.$$

Here $R^{12}=R\otimes\mathrm{id}$ etc. and V is a finite- (or infinite-)dimensional linear space over the field $k=\mathbb{R}$ or $\mathbb{C}$. If $R=\sigma\cdot S$, where σ is the permutation $\sigma(v_1\otimes v_2)=v_2\otimes v_1$, $v_i\in V$, then QYBE can be rewritten as follows:

$$(S\otimes\mathrm{id})(\mathrm{id}\otimes S)(S\otimes\mathrm{id})=(\mathrm{id}\otimes S)(S\otimes\mathrm{id})(\mathrm{id}\otimes S). \qquad (0.1)$$

Thus, an R-matrix in the form (0.1) gives a "local" representation of the braid group. Henceforth we call an operator S satisfying (0.1) a YB operator. And we call a YB operator a symmetry if it satisfies the "unitarity" condition $S^2=\mathrm{id}$ (or $R^{12}\cdot R^{21}=\mathrm{id}$, where $R^{21}=\sigma R^{12}\sigma$ for R).

A classical analogue of this object is a "classical R-matrix", i.e. an operator $R: V^{\otimes 2}\to V^{\otimes 2}$ satisfying the equation

Elsevier Science Publishers B.V.