Hele–Shaw flows with nonlinear kinetic undercooling regularization

Nikolaij Borisovich Pleshchinskii[a], Michael Reissig[b],*

[a]*Department of Applied Mathematics, Kazan State University, 420008 Kazan, Russia*
[b]*Faculty of Mathematics and Computer Science, TU Bergakademie Freiberg, Agricola Str. 1, D-09596 Freiberg, Germany*

1. Introduction

The first exact solutions for classical Hele–Shaw flows driven by a single sink at the origin were constructed in [3,8]. They have finite-time blow-up and cusp formation at the free boundary before it reaches the sink (see e.g. [6]). Consequently, the classical Hele–Shaw model is globally ill-posed in this respect. The physical meaning of this situation is that the velocity of points on the moving boundary $\Gamma(t)$ tends to infinity at the cusp.

Among different approaches to regularize the suction problem there exists one to incorporate an extra term in the dynamic condition at the free boundary to penalize large normal derivatives. This regularization is called kinetic undercooling regularization. For Hele–Shaw flows it was proposed in [12]. The idea is to include a kinetic undercooling term proportional to the normal velocity at the moving boundary.

This idea was used in several papers [5,9,11] and the linear dynamic condition

$$p + \alpha \frac{\partial p}{\partial n} = 0 \quad \text{on } \Gamma(t) \tag{1.1}$$

---

* Corresponding author.

E-mail addresses: nikolai.pleshchinskii@ksu.ru (N.B.Pleshchinskii); reissig@math.tu-freiberg.de (M.Reissig).