

Marcinkiewicz-type law of large numbers for double arrays

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Abstract

Chatterji strengthened version of a theorem for martingales which is a generalization of a theorem of Marcinkiewicz proving that if X_n is a sequence of independent, identically distributed random variables with $E|X_n|^p < \infty$, $0 < p < 2$ and $EX_1 = 0$ if $1 \leq p \leq 2$, then $n^{-1/p} \sum_{i=1}^n X_i \rightarrow 0$ a.s. and in L_p . In this paper, we prove a version of law of large numbers for double arrays. If $\{X_{ij}\}$ is a double sequence of random variables with $E|X_{11}|^p \log^+ |X_{11}|^p < \infty$, $0 < p < 2$, then $\lim_{m, n \rightarrow \infty} m^{-1/p} \sum_{i=1}^m \sum_{j=1}^n (X_{ij} - a_{ij}) / (mn)^{1/p} = 0$ a.s. and in L_p , where $a_{ij} = 0$ if $0 < p < 1$, and $a_{ij} = E[X_{ij}|F_{ij}]$ if $1 \leq p \leq 2$, which is a generalization of Etemadi's Marcinkiewicz-type SLLN for double arrays. This also generalizes earlier results of Smythe, and Gut for double arrays of i.i.d. r.v.'s.

Keywords

Double arrays, L_p convergence, Strong law of large numbers