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Dynamic capillary pressure mechanism for instability in gravity-driven flows; review and extension to very dry conditions

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Abstract

Several alternative mathematical models for describing water flow in unsaturated porous media are presented. These models are based on an equation for conservation of mass of water, and a generalized linear law for water flux (Darcy's law) containing a term called the dynamic capillary pressure. The distinct form of each alternative model is based on the specific form of expression used to describe the dynamic capillary pressure. The conventional representation arises when this pressure is set equal to the equilibrium pressure given by the capillary pressure - saturation function for unsaturated porous media, and this conventional approach leads to the Richards equation. Other models are derived by representing the dynamic capillary pressure by a rheological relationship stating that the pressure is not given directly by the capillary pressure - saturation function. Two forms of rheological relationship are considered in this manuscript, a very general non-equilibrium relation, and a more specific relation expressed by a first-order kinetic equation referred to as a relaxation relation. For the general non-equilibrium relation the system of governing equations is called the general Non-Equilibrium Richards Equation (NERE), and for the case of the relaxation relation the system is called the Relaxation Non-Equilibrium Richards Equation (RNERE). Each of the alternative models was analyzed for flow characteristics under gravity-dominant conditions by using a traveling wave transformation for the model equations, and more importantly the flow described by each model was analyzed for linear stability. It is shown that when a flow field is perturbed by infinitesimal disturbances, the RE is unconditionally stable, while both the NERE and the RNERE are conditionally stable. The stability analysis for the NERE was limited to disturbances in the very low frequency range because of the general form of the NERE model. This analysis resulted in what we call a low-frequency criterion (LFC) for stability. This LFC is also shown to apply to the stability of the RE and the RNERE. The LFC is applied to stability analysis of the RNERE model for conditions of initial saturation less than residual.

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