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On condensed forms for partially commuting matrices

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Abstract

Two complex $n \times n$ matrices A and B are said to be partially commuting if A and B have a common eigenvector. We propose a condensed form for such matrices that can be obtained from A and B by a finite rational computation. The condensed form is a pair of block triangular matrices, with the sizes of the blocks being uniquely defined by the original matrices. We then show how to obtain additional zeros inside the diagonal blocks of a condensed form by using the generalized Lanczos procedure as given by Elsner and Ikramov. This procedure can also be considered as a finite rational process. We point out several applications of the constructions above. It turns out that for Laffey pairs of matrices, i.e., for matrices (A, B) such that $\operatorname{rank}[A, B] = 1$, the condensed form is a pair of 2×2 block triangular matrices. Using this fact, we show an economical way to find a spanning set for the matrix algebra generated by Laffey matrices A and B. Another application concerns so-called k-self-adjoint matrices. We examine such matrices in the unitary space as well as in a Krein space of defect 1. As an Appendix, we give a new description of the Shemesh subspace of matrices A and B. This is the maximal common invariant subspace of A and B, on which these matrices commute.

Keywords: Condensed forms; Invariant subspaces; Partially commuting; Finite rational process

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