Sharp Hardy type inequalities with weights depending on bessel function

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Abstract

© Nasibullin R.G. 2017. We prove exact Hardy type inequalities with the weights depending on a Bessel function. We obtain one-dimensional L_p-inequalities and provide an example of extending these inequalities for the case of convex domains with a finite inner radius. The proved statements are generalization for the case of arbitrary $p \geq 2$ of the corresponding inequality proved by F.G. Avkhadiev and K.-J. Wirths for $p = 2$.

Keywords

Bessel function, Convex domains, Distance function, Hardy inequality, Inner radius, Lamb constant

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