

Differential calculus on h -deformed spaces

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Abstract

© 2017, Institute of Mathematics. All rights reserved. We construct the rings of generalized differential operators on the h -deformed vector space of gl -type. In contrast to the q -deformed vector space, where the ring of differential operators is unique up to an isomorphism, the general ring of h -deformed differential operators $\text{Diff } h, \sigma(n)$ is labeled by a rational function σ in n variables, satisfying an over-determined system of finite-difference equations. We obtain the general solution of the system and describe some properties of the rings $\text{Diff } h, \sigma(n)$.

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Keywords

Differential operators, Poincaré-Birkhoff-Witt property, Reduction algebras, Representation theory, Rings of fractions, Universal enveloping algebra, Yang-Baxter equation

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