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Eigenvibrations of a Beam with Load

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Abstract—The eigenvalue problem describing eigenvibrations of a beam with load is investigated. The problem has an increasing sequence of positive simple eigenvalues with limit point at infinity. To the sequence of eigenvalues, there corresponds a system of normalized eigenfunctions. Limit properties of eigenvalues and eigenfunctions are studied.

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1. INTRODUCTION

Let us formulate the differential eigenvalue problem governing eigenvibrations of the beam-load system. Assume that the beam axis occupies in the equilibrium horizontal position the segment $[0,l]$ on the Ox axis. Denote by $\rho(x)$ the volume mass density, $E(x)$ the elasticity Young modulus, $S(x)$ the square of transversal cut of the beam, $J(x)$ the inertia moment of the cut with respect to its horizontal axis at the point $x \in [0,l]$. Suppose that the end $x=0$ of the beam is rigidly fixed while the end $x=l$ is free, at the point $x=l$ of the beam a load of mass M is rigidly joined. Then the vertical deflection $w(x,t)$ of the beam at a point x at time t satisfies the following equations

$$\frac{\partial^2}{\partial x^2}\left(p(x)\frac{\partial^2}{\partial x^2}w(x,t)\right) + r(x)\frac{\partial^2}{\partial t^2}w(x,t) = 0, \quad x \in (0,l), \quad t > 0, \tag{1}$$

$$w(0,t) = \frac{\partial}{\partial x}w(0,t) = 0, \quad t > 0, \tag{2}$$

$$\frac{\partial^2}{\partial x^2}w(l,t) = \frac{\partial}{\partial x}\left(p(l)\frac{\partial^2}{\partial x^2}w(l,t)\right) - M\frac{\partial^2}{\partial t^2}w(l,t) = 0, \quad t > 0, \tag{3}$$

where $p(x) = E(x)J(x)$, $r(x) = \rho(x)S(x)$, $x \in [0,l]$.

The eigenvibrations of the beam-load system are characterized by the function $w(x,t)$ of the form $w(x,t) = u(x)v(t)$, $x \in [0,l]$, where $v(t) = a_0\cos\sqrt{\lambda}t + b_0\sin\sqrt{\lambda}t$, $t > 0$; a_0, b_0, and λ are constants. Then equations (1)–(3) lead to the following eigenvalue problem: find values λ and nontrivial functions $u(x)$, $x \in [0,l]$, such that

$$(p(x)u''(x))'' = \lambda r(x)u(x), \quad x \in (0,l), u(0) = u'(0) = 0, \quad u''(l) = (p(l)u''(l))' + \lambda M u(l) = 0. \tag{4}$$

Problem (4) has an increasing sequence of positive simple eigenvalues with limit point at infinity. To the sequence of eigenvalues, there corresponds a system of normalized eigenfunctions. In the present

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