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# The analytical and numerical approaches to the theory of the Moon's librations: Modern analysis and results

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## Abstract

Observing physical librations of celestial bodies and the Moon represents one of the astronomical methods of remotely assessing the internal structure of a celestial body without conducting expensive space experiments. The paper contains a review of recent advances in studying the Moon's structure using various methods of obtaining and applying the lunar physical librations (LPhL) data.

In this article LPhL simulation methods of assessing viscoelastic and dissipative properties of the lunar body and lunar core parameters, whose existence has been recently confirmed during the seismic data reprocessing of “Apollo” space mission, are described. Much attention is paid to physical interpretation of the free librations phenomenon and the methods for its determination.

In the paper the practical application of the most accurate analytical LPhL tables (Rambaux and Williams, 2011) is discussed. The tables were built on the basis of complex analytical processing of the residual differences obtained when comparing long-term series of laser observations with the numerical ephemeris DE421.

In the paper an efficiency analysis of two approaches to LPhL theory is conducted: the numerical and the analytical ones. It has been shown that in lunar investigation both approaches complement each other in various aspects: the numerical approach provides high accuracy of the theory, which is required for the proper processing of modern observations, the analytical approach allows to comprehend the essence of the phenomena in the lunar rotation, predict and interpret new effects in the observations of lunar body and lunar core parameters.

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## 1. Introduction the role of celestial bodies' physical librations investigation in modern celestial mechanics

Observing physical librations of celestial bodies and the Moon represents an astronomical method of remote assessing the internal structure of a celestial body without conducting expensive space experiments. For instance, at the

beginning of the first decade of the 21st century there were a series of articles (Gudkova and Zharkov, 2006; Jolliff et al., 2006; Van Hools et al., 2007; Mocquet et al., 2011) clearly indicating a new direction in the study of celestial bodies: the internal structure assessing and its subsequent simulation are bound to observations of rotation of the Moon, Jupiter's, and Saturn's moons.

In the new decade a stream of articles on the application of observations of rotation features of Saturn and Enceladus (Lainey et al., 2012, 2017), Titan (Baland et al., 2012), and of course the Moon (Rambaux and Williams, 2011; Barkin et al., 2014; Gusev et al., 2015)

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