# Classical Yang-Baxter equation from supergravity

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## Abstract

© 2018 authors. Published by the American Physical Society. Published by the American Physical Society under the terms of the »https://creativecommons.org/licenses/by/4.0/» Creative Commons Attribution 4.0 International license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP 3 . We promote the open-closed string map, originally formulated by Seiberg & Witten, to a solution generating prescription in generalized supergravity. The approach hinges on a knowledge of an antisymmetric bivector Θ, built from antisymmetric products of Killing vectors, which is specified by the equations of motion. In the cases we study, the equations of motion reproduce the classical Yang-Baxter equation (CYBE) and Θ is the most general r-matrix solution. Our work generalizes Yang-Baxter deformations to non-coset spaces and unlocks gravity as a means to classify r-matrix solutions to the CYBE.

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