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Unified approach to the entropy of an extremal rotating BTZ black hole: Thin shells and horizon limits

Lemos J., Minamitsuji M., Zaslavskii O.
Kazan Federal University, 420008, Kremlevskaya 18, Kazan, Russia

Abstract

© 2017 American Physical Society. Using a thin shell, the first law of thermodynamics, and a unified approach, we study the thermodymanics and find the entropy of a (2+1)-dimensional extremal rotating Bañados-Teitelbom-Zanelli (BTZ) black hole. The shell in (2+1) dimensions, i.e., a ring, is taken to be circularly symmetric and rotating, with the inner region being a ground state of the anti-de Sitter spacetime and the outer region being the rotating BTZ spacetime. The extremal BTZ rotating black hole can be obtained in three different ways depending on the way the shell approaches its own gravitational or horizon radius. These ways are explicitly worked out. The resulting three cases give that the BTZ black hole entropy is either the Bekenstein-Hawking entropy, S=A+4G, or an arbitrary function of A+, S=S(A+), where A+=2πr+ is the area, i.e., the perimeter, of the event horizon in (2+1) dimensions. We speculate that the entropy of an extremal black hole should obey 0≤S(A+)≤A+4G. We also show that the contributions from the various thermodynamic quantities, namely, the mass, the circular velocity, and the temperature, for the entropy in all three cases are distinct. This study complements the previous studies in thin shell thermodynamics and entropy for BTZ black holes. It also corroborates the results found for a (3+1)-dimensional extremal electrically charged Reissner-Nordström black hole.

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