IMECE2011-62869

ALGORITHM OF LAUNCHED VEHICLES MASS CALCULATION AT THE EARLY STAGE OF DESIGNING

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ABSTRACT

Launching mass of pilotless vehicle bounds with flight path. Required fuel mass depends from flight trajectory. Design calculation of vehicle mass and flight trajectory are in close bind. We suggest numerical algorithm of design calculation of vehicle flight mass and optimal characteristics of flight trajectory – initial flight-path angle and flight time to target.

Keywords: calculation of vehicle mass, optimal flight-path angle, flight time to target.

INTRODUCTION

Specific masses of wing, fuselage, fuel system, engine, equipment etc. of certain vehicle type have narrow range of values. We can use statistic date. It allows us to calculate optimal characteristics of flight trajectory and minimum volume of required fuel mass. Vehicle's flight mass depends from flight time to target, initial flight-path angle, specific engine impulse, limit load factor and limit flight velocity variation.

Pilotless vehicle may be multistage structure. Mass of finale stage vehicle m_N can be expressed by following formula:

$$m_N = \left[m_{p.l.}(1+\varphi_{p.l.})\right] / \left[1-\overline{m}_{sN}-(1+\delta_N)\overline{m}_{fN}\right], \quad (1)$$

where $m_{p.l.}$ – payload;

$\varphi_{p.l.}$ – coefficient that token in to account fuselage mass, required for payload placing;

$\overline{m}_{sN}$ – specific mass of finale stage vehicle (fuselage, wings, tail, engine, control system);

$\overline{m}_f$ – fuel specific mass;

δ – coefficient that token in to account mass of fuel system. Specific mass $\overline{m}_s$ and coefficient δ are aggregative volumes in general case – they include specific mass of plane elements and elements of fuel system. Mass of vehicle finale stage is payload for next-to-last vehicle stage itc.

General formula of Launching mass in the first approximation is:

$$m_N = \left[m_{p.l.}(1+\varphi_{p.l})\right] / \prod_{i=1}^{N}\left[1-\overline{m}_{si}-(1+\delta_i)\overline{m}_{fi}\right], \quad (2)$$

where symbol $\prod_{i=1}^{N}$ - means multiplication.

DESCRIPTION OF NUMERICAL TRAJECTORY CALCULATION

The first we define equation of three dimensional trajectory. Divide flight time T_0 into *n-1* intervals $T_i = T = T_0/(n-1)$. Define equation of three dimensional trajectory segment between points *i-1* and *i* by following cubic function:

$$\mathbf{r}(t) = \mathbf{r}_{i-1} + \dot{\mathbf{r}}_{i-1}t + \ddot{\mathbf{r}}_{i-1}\left(\frac{t^2}{2}-\frac{t^3}{6T}\right) + \ddot{\mathbf{r}}_i\frac{t^3}{6T}, \quad (3)$$

where $\mathbf{r}(t)$ - is current radius-vector;

$\mathbf{r}_{i-1}$, $\dot{\mathbf{r}}_{i-1}$ and $\ddot{\mathbf{r}}_{i-1}$ are radius–vector, the first and second order derivative in the point *i-1*correspondently.

Bind $\mathbf{r}_n$ and $\dot{\mathbf{r}}_n$ with $\ddot{\mathbf{r}}_i$ by following matrix equation

$$\begin{bmatrix} n/2-2/3 & n-2 & n-3 & \dots & 2 & 1 & 1/6 \\ 1/2 & 1 & 1 & \dots & 1 & 1 & 1 \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{r}}_1 \\ \dots \\ \ddot{\mathbf{r}}_n \end{Bmatrix} + $$

$$+ \begin{Bmatrix} \dfrac{\mathbf{r}_1-\mathbf{r}_n}{T^2} + \dfrac{(n-1)}{T}\dot{\mathbf{r}}_1 \\ \dfrac{\dot{\mathbf{r}}_1-\dot{\mathbf{r}}_n}{T} \end{Bmatrix} = 0, \quad (4)$$

and introduce following term:

$$L=\frac{1}{2}\left(\ddot{\mathbf{r}}_1,\ldots,\ddot{\mathbf{r}}_n\right)[Q]\left(\ddot{\mathbf{r}}_1,\ldots,\ddot{\mathbf{r}}_n\right)^T\to\min, \qquad (5)$$

where $[Q]$ – is some diagonal matrix. In case of unit matrix Q clause (2) corresponds minimum of sum $\sum_{i=1}^{n}\left(\ddot{\mathbf{r}}_i\right)^2$.

Equations (4), (5) are allowed us to define three dimensional trajectory. We can set clause $\ddot{\mathbf{r}}_n\to 0$ or select desirable spread character of $\ddot{\mathbf{r}}_i$ by matrix $[Q]$.

Lets define segment of three dimensional trajectory for initial angle of path θ_1 (0, 30, 60, 90 degree) by using equations (3)-(5) and term $\ddot{\mathbf{r}}_n\to 0$. Preset following date:

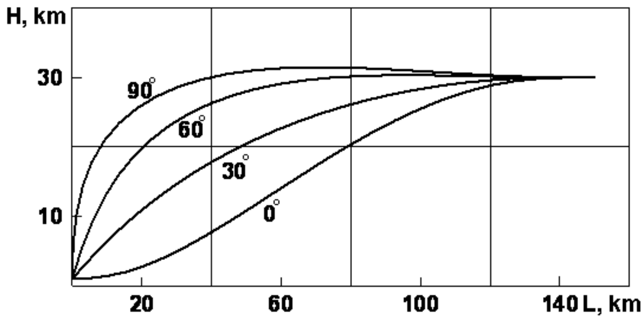

Fig.1

initial velocity $V_1 = 800$ *m/s*;
finale velocity $V_n = 1200$ *m/s*;
initial height $h_1 = 1$ *km* finale height $h_n = 30$ *km*; range $\Delta L = 150$ *km*; flight time to target $T_0 = 150$ *s*. In fig.1 flight paths are shown.

NUMERICAL ALGORITHM LAUNCHED PATH ANGLE, WEIGHT, FLIGHT TIME AND SPECIFIC FUEL MASS CALCULATION

Equation of advanced motion of vehicle's center of gravity (C–of–G) in tangent exis of wind-body coordinate system is [1]

$$m\dot{V}=P\cos\alpha-X_a-mg\sin\theta, \qquad (6)$$

where $\dot{V}=dV/dt$; m – is vehicle flight mass; α - is angle of attack; P – is thrust; X_a - is drag; θ – is flight path angle.

Thrust $P(t)$ can be defined through specific impulse $I(h)$ и fuel consumption per second: $P=-(dm/dt)\,gI(h)$. Transform equation (6) to numerical calculation

$$m_{(i+1)}=m_{(i)}+\left(\frac{dm}{dt}\right)_{(i)}\Delta t, \qquad (7)$$

$$\left(\frac{dm}{dt}\right)_{(i)}=-\left[m_{(i)}\dot{V}_i/g+m_0\left[\sigma_{(i)}+\sigma_{a(i)}\right]q_{(i)}+m_{(i)\_}\sin\theta_{(i)}\right]/I(h_i)\cos(\alpha_i)$$

$(i=1,n)$,

where Δt – is time step ;
n – is number of discrete points;
σ and σ_a – ballistic coefficients of parasitic and inductive drag;
$q=\rho V^2/2$ – velocity (dynamic) pressure.

Ballistic coefficient σ is defined by equation [2]

$$\sigma q=\frac{X_a}{m_0 g}\ . \qquad (8)$$

If Lift is zero and Makh number $M > 1$ we can define σ through coefficient D and launched vehicle weight G_0 [2]

$$\sigma_0 = D/G_0^{1/3}. \qquad (9)$$

Coefficient D depends of vehicle type and type of engine, vehicle with rocket engine has minimum of D volumes; vehicle with jet engine has maximum volumes of D. We can define coefficients D compare similar vehicles with similar fuselage shape and the same type of engine. Ballistic coefficient σ decreases when Makh-number increases for vehicle with sharpened nose: $\sigma(M)=\sigma_0(A+\frac{B}{M})$,

A and *B* depend of vehicle's stream–line form [2]. Volume *A* decreases and volume *B* increases for elongated and sharpened nose if compare with blunt short vehicle's nose. For ballistic rockets $A = 0{,}31$; $B = 1{,}73$ ($M = 2{,}5$). Usually accepted $A = 0{,}5$ и $B = 1{,}25$. Coefficient D is *1,5-2* times bigger for jet-propelled vehicles. Coefficient $C_x=C_x(M)$ or $\sigma=\sigma(M)$ alters slightly if vehicle has short blunt nose, $A\approx 1$, $B\approx 0$. In case of subsonic velocity $\sigma_{0M<1}=(0{,}6-0{,}7)\sigma_0$.

Ballistic coefficient of induced drag is given by following expression:

$$\sigma_\alpha=\frac{C_{xi}S}{G_0}=\frac{C_{xi}}{p_0}, \qquad (10)$$

where C_{xi} – is coefficient of induced drag:

$$C_{xi} = kC_y^2 \text{ or } \sigma_a = k\frac{C_y^2}{p_0}. \qquad (11)$$

In case of supersonic velocity [2]

k = 0,08 + 0,23M

(for *M > 2; 2 >* $\lambda_{кр}$ *> 0,7;* η *>3*), (12)

where $\lambda_{кр}$ and η – are wing aspect ratio and wing taper correspondently.

In case of subsonic velocity [3]

$$k = \frac{1}{\pi\lambda_{эфф}}, \text{ where}$$

$$\lambda_{эфф} = \frac{\lambda}{1 + 0{,}0314\lambda\cdot\cos^2\chi}, \qquad (13)$$

where χ – is wing sweep angle.

Suppose slip angle $\beta = 0$, cross-wind aerodynamic force $Z_a = 0$ and write another two equations of advanced motion of vehicle's center of gravity (C–of–G) in wind-body coordinate system [1]

$$\begin{aligned} mV\dot{\theta} &= P\sin\alpha\cos\gamma_a + Y_a\cos\gamma_a - mg\cos\theta \\ mV\dot{\Psi}\cos\theta &= P\sin\alpha\sin\gamma_a + Y_a\sin\gamma_a, \end{aligned} \qquad (14)$$

where γ_a – is angle of roll, Ψ – is course angle; Y_a – is lift.

Equations (14) is used for Y_a calculation. Angle of attack α is calculated by following approximate expression of $\partial C_y / \partial\alpha$ for subsonic and supersonic velocity [2], [3]:

$$C_y^\alpha = \frac{6{,}076}{\left(\frac{1}{\cos\chi} + \frac{2}{\lambda}\right)\sqrt{1-M^2}} \quad (M<1);$$

$$C_y^\alpha = \frac{4}{\sqrt{M^2-1}}\left(1 - \frac{1}{2\lambda\sqrt{M^2-1}}\right) \quad (M>1)$$

(15)

Suppose $\varphi_{p.l.} = 0{,}3$;

$\overline{m}_S = \overline{m}_{fuz} + \overline{m}_{wing} + \overline{m}_{Eng} = 0{,}06 + 0{,}06 + 0{,}03 = 0{,}15$

(fuselage, wings, engine); $\overline{m}_T = 0{,}4$;

$\alpha = 0{,}18$– for fuel tanks. Calculate previously the launching mass in a first approximation $G_0 = 17460$ N.

Volume of $\sigma_0 = 0{,}669\times10^{-4}$ can be obtained by formula (9), where $D = 8{,}0\times10^{-4}$ (statistic data for rocket engine vehicles). Take into account that variation of specific impulse *I* not exceed 11% and accept linear law changing of impulse through the flight height:

$$I(h) = I_1\left(1 - \frac{h}{\Delta H}\right) + I_n\frac{h}{\Delta H}.$$

Set volume of specific wing load $p_0 = \frac{G_0}{S} \approx 2400\ N/m^2$ (statistic date), payload $G_{p.l.} = 5000N$;

specific impulse $I_{1(h=1km)} = 240s$;

$I_{n(h=15km)} = 270s$ (rocket engine).

After calculation of m_n in the last point of trajectory by integrating of equation (7) we can obtain $\overline{m}_T = (m_0 - m_n)/m_0$. Then we may calculate G_0, σ_0 anew and recalculate $\overline{m}_T$. We have do it while solution convergence is got. If we calculate G_0, $\overline{m}_T$ for several initial path angle θ we reveal that there is certain angle θ_1 that delivered minimum of G_0, $\overline{m}_T$. Lets find this initial path angle.

At the first consider one-parameter searching of G_0 minimum that delivered by flight time T_0. Preset initial $T_0 = 150s$. Then calculate $G_0(T_0)$, $G_0(T_0 - \Delta T)$, $G_0(T_0 + \Delta T)$ and define new volume of T_0, by known Newton scheme

$$T_{0(i)} = T_{0(i-1)} + \\ + \varphi\frac{\Delta T[G_0(T_{0(i-1)} - \Delta T) - G_0(T_{0(i-1)} + \Delta T)]}{G_0(T_{0(i-1)} - \Delta T) - 2G_0(T_{0(i-1)}) + G_0(T_{0(i-1)} + \Delta T)}, \qquad (16)$$

where $\varphi < 1$ – some coefficient ($\varphi \approx 0{,}5$);

i – number of iteration.

We can calculate T_0 for several volumes of θ_1 from $\theta_1 = 0^0$ to $\theta_1 = 90^0$.

In fig.2, fig.3, fig.4 are shown variations of flight time *T*₀, launched vehicle mass G_0 (fig.3) and specific fuel mass $\overline{m}_T$ correspondently versus launched path angle θ_1.

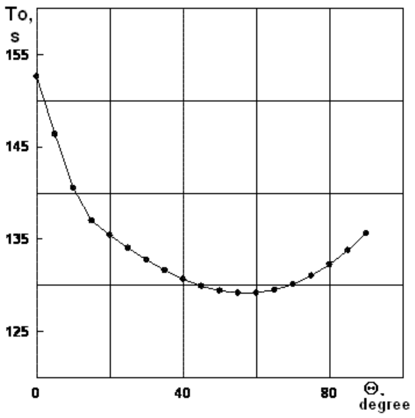

Fig.2

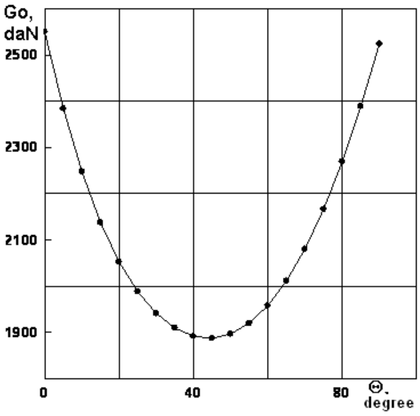

Fig.3

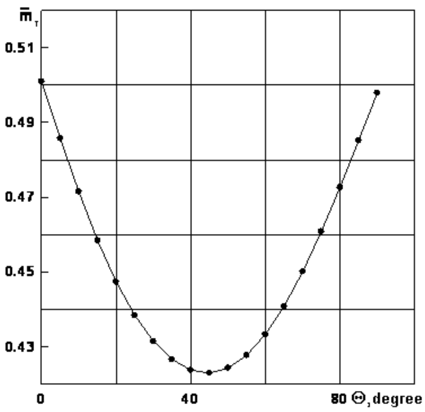

Fig.4

There is minimum of vehicle mass G_0 as we can see in fig.3 and fig.4. Try to find volumes of θ_1, *T0*, that delivered minimum vehicle mass G_0. Join parameters θ_1, *T0* and write its in matrix form $\mathbf{x} = (\theta_{1,} T_0)^T$. We use following discrete form of Newton scheme for searching of $G(\mathbf{x})$ minimum

$$\mathbf{x}_{i+1} = \mathbf{x}_i - \varphi\left[d^2 G_0(\mathbf{x}) / d\mathbf{x}^2\right]_i^{-1} dG_0(\mathbf{x}) / d\mathbf{x}, \quad (17)$$

$$dG(\mathbf{x}) / dx_j \approx \left(G_0(x_j + \Delta x_j) - G_0(x_j - \Delta x_j)\right) / 2\Delta x_j;$$

$$d^2 G_0(\mathbf{x}) / dx_k dx_j \approx$$

$$\approx \left[G_0(x_k + \Delta x_k, x_j + \Delta x_j) - G_0(x_k + \Delta x_k, x_j - \Delta x_j) - \right.$$

$$\left. - G_0(x_k - \Delta x_k, x_j + \Delta x_j) + G_0(x_k - \Delta x_k, x_j - \Delta x_j)\right] / 4\Delta x_k \Delta x_j$$

k,j=1,2

Finale we obtain: flight time $T_0 = 130s$; launched path angle $\theta_{1.} = 44{,}3^O$; launched vehicle weight $G_{0.} = 18879\ N$; specific fuel mass $\overline{m}_{f\,\min.} = 0{,}423$ for preset specific engine impulse $I_{1(h=1km)} = 240s$, $I_{n(h=15km)} = 270s$.

Now we may calculate changing of thrust, aerodynamic lift and drag by using equations (7), (14).

In fig.5 dependences $P(t)$, $X_a(t)$, $G(t)$, $Y_a(t)$ (curves 1, 2, 3, 4 correspondently) are shown. This results can be used in design calculation of wing, tail geometry. We can calculate parameters T_0, $\theta_{1.}$, $G_{0.}$, $\overline{m}_f$. anew after ascertaining of aerodynamic characteristics in the second approximation.

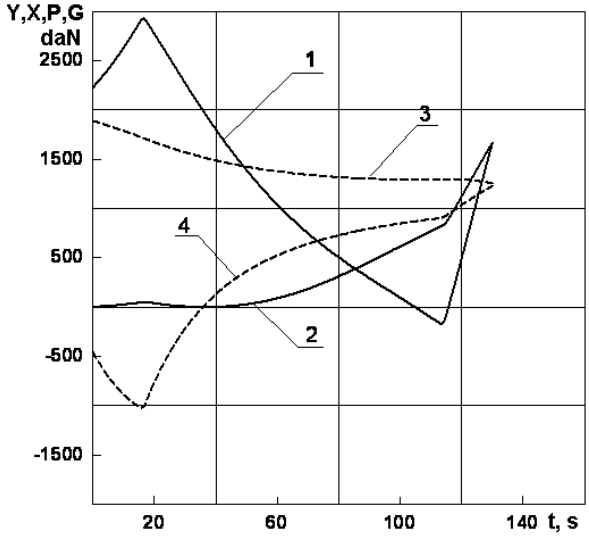

Fig.5

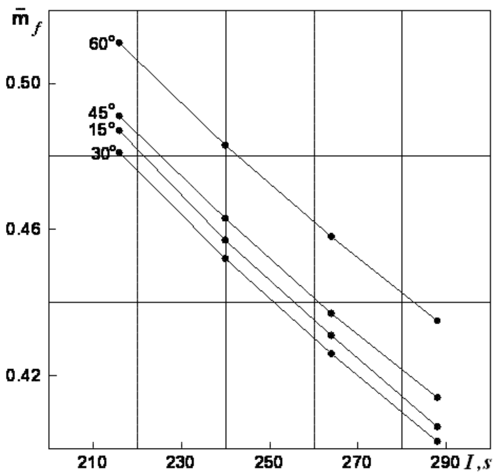

Fig.6

Dependences specific fuel mass $\overline{m}_f$ versus specific engine impulse I are shown in fig.6. We can see that dependence $\overline{m}_f(I)$ is linear. Its allows us to modify formula (2) and express it in following form:

$$m_N = \left[m_{p.l}(1+\varphi_{p.l.})\right] / \prod_{i=1}^{N}\left[1-\overline{m}_{si}-(1+\delta_i)(\overline{m}_{fi}+k_i\Delta I_i)\right], \tag{18}$$

where ΔI – is increment of specific impulse of modified vehicle if compare with known vehicle. In our case $k = -1{,}07\times10^{-3}$.

CONCLUSIONS

Suggested numerical algorithm of vehicle characteristic's optimization may be useful in design calculation at the early stage of designing, because first stage results have a big influence to choosing of design parameter.

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