In the manuscript

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RESEARCH OF THE SOLVABILITY OF BOUNDARY PROBLEMS FOR OPERATOR-DIFFERENTIAL EQUATIONS OF MIXED TYPE

01.01.02 – Differential equations, dynamical systems, and optimal control

ABSTRACT
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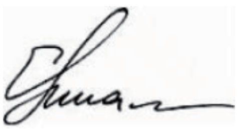
Abstract sent «____» «______________» 2016 г.



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Relevance of the topic. Let E – complex Hilbert space with inner product $(\cdot,\cdot)$ and the norm $||\cdot||$, and B, L – linear operators acting in it. Consider the equation

$$Bu_t = Lu + f, \quad t \in (0,T), \quad (T \leq \infty). \tag{1}$$

Boundary value problem for the equation (1) is an abstract form many boundary value problems for equations and systems of equations in partial to integral-differential equations. Even in the simplest class of equations includes a significant number of problems arising in mathematical physics.

The study of the equation (1) Sobolev type in different cases space E and the operator B, L engaged many mathematicians, among them Weierstrass, Kronecker, F.R. Gantmakher, J.E. Boyarintsev, Mathematics from school S.G. Crane, V.B. Osipov, S.D. Adelman, S.P. Zubov, K.I. Chernyshev, R.E. Showalter, A.G. Rutkas, N.I. Radbel, N.A. Sidorov, M.V. Falaleev, A.I. Kozhanov, A. Favini, G.A. Sviridyuk, V.E. Fedorov, I.V. Melnikova, M.A. Alshansky and others.

The equation of the form (1), which is not a Sobolev-type equation in abstract form was studied, for example, R. Beals, NV Kislov, B. Greenberg, K.V.M. van der Mee, P.F. Zveyfel, S.G. Pyatkov, P. Grisvard. Among the methods used to study the solvability of boundary value problems for these equations can be highlight the variational method based on projection theorems Lax-Milgram, methods of group theory, Fourier method (Eigenfunction expansion).

For equations of Sobolev type or close to them, and also for some equations that do not belong to the Sobolev type, correct regular Cauchy problem or task that is close to it. other situation if no equation is an equation of type Sobolev (usually, this means that the spectrum of the operator B contains both positive and infinite subset negative real axis). Earlier in the works M.S. Baouendi, R. Beals, M. Gevrey, N.V. Kislov, S.D. Pagani, S.A. Tersenov were studied correct boundary value problems for model equations of the form (1). In this case, study questions of solvability, uniqueness and stability making a number of problems, mainly related to the fact, at this time interval the solution of this problem is not always exist. As a rule, it exists (for example, the decision of the initial boundary value problem), but at some small time gap, and then can disintegrate in the sense that the solution or derivatives thereof may apply to the ∞. An example be the case when the coefficients of the equation at some surface in the job equation misbehave, for example, refer to ∞.

The purpose of the work. The main purpose is the proof of existence and uniqueness theorems, study of the properties of solutions of local and nonlocal boundary value problems for operator-differential equations of mixed type, as well as study Application of the results for the model equations odd order with changing time direction.

Methods. In the study of local and nonlocal boundary value problems for operator-differential equations of mixed such as the techniques and methods used were developed for tasks initial data, soluble in the whole time. Here first all should be allocated monograph O.A. Ladyzhenskaya, V.A. Solonnikov, N.I. Uraltseva (1967), as well as monographs S.G. Pyatkov

(2000, 2002). It uses methods of functional analysis methods of the theory of partial differential equations derivatives.

In the proof of the existence of the desired solution to the problem Gevrey third-order equation using the potential method by which the boundary value problem is reduced to the investigation of a system of singular integral equations with respect to which we note monograph N.F. Gakhov (1963), N. Muskhelishvili (1962), L.G. Mikhailov (1966), T.D. Dzuraev (1979).

Scientific novelty. The main obtained in the thesis results:

• investigated marginal Gevrey-type problems for new classes of operator-differential equations of mixed type with an arbitrary dissipative operator in the main part;

• theorems of existence of a generalized solution, studied the smoothness of solutions in weighted Sobolev spaces, and the applications of the results to the equations of odd order with changing time direction;

• proved the solvability of a wide class of nonlocal boundary value problems for operator-differential equations of mixed type with an operator satisfying the condition Kato-sectoriality in the main part, and the question of the smoothness of the solutions of these problems in weighted Sobolev spaces;

• for a third-order equations with one spatial variable with multiple characteristics theorems of solvability in Holder spaces of boundary problems such as Gevrey.

All results are new.

Testing of work. The results obtained in the thesis were reported and discussed at the Seminar at the Department of Differential Equations of Kazan (Volga) Federal University under the direction of D.Sc., Professor V.I. Zhegalov (Kazan 2015) at the joint seminar of the department of mathematical analysis NEFU (Yakutsk: 2014, 2015), research institutes Mathematics NEFU «Nonclassical differential equations, controlled processes and applications» (director Dr., Professor I.E. Egorov), on XLVII–XLIX International scientific student conferences « Student and scientific and technical progress » (Novosibirsk: 2009–2012); to XIX, XXI International Conference of Students and young scientists «Lomonosov» (2012, 2014: Moscow) at the III Russian Scientific Conference and VII Russian School-Seminar students, graduate students, young scientists and Professionals «Mathematical modeling of the Nordic Russian territories» (Yakutsk: 2012); at the International Conference dedicated to the 80th anniversary of from the date of the birth of Academician M.M. Lavrentyev «Inverse and ill-posed problems of mathematical physics » (Novosibirsk: 2012); at the IV International Youth Scientific School-Conference «Theory and Computational Methods for Inverse and ill-posed problems » (Novosibirsk: 2012); at the International Conference dedicated to the 105th anniversary of the birth of S.L. Sobolev (Novosibirsk: 2013); at the VII International Conference on Mathematical Modelling (Yakutsk: 2014); at the International Conference «Differential equations and mathematical modeling» (Ulan-Ude: 2015).

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Publication. The main results of the thesis were published 25 works of the author: 8 articles [1–8], 17 abstracts [9–25].

In a joint paper [7] setting goals, theorems, proofs idea solvability of boundary value problems belong to S.G. Pyatkov.

8 articles [1–8] published in journals from the list of peer-reviewed scientific publications HAC, including article 3 [5–7] (1 article translated) are included in the international database of abstracts and citation systems Web of Science, Scopus.

The structure and scope of work. The thesis consists of an introduction, three chapters, containing 8 paragraphs, conclusion and bibliography. Total volume is 112 pages. References contains 139 names.

Summary of the thesis

In the introduction the urgency of the dissertation topics given the historical information on the topic of the thesis, and briefly describes the content of the work.

In the first chapter, consisting of three sections, we consider boundary value problems for operator-differential equations of the form

$$Au \equiv Bu_t - Lu = f(x,t), \tag{2}$$

where the linear operators B, L defined in this Hilbert space E, and the operator B is self-adjoint. Boundary conditions are

$$P^+u(0) = u_0, \quad P^-u(T) = u_T, \tag{3}$$

where P^+, P^- — spectral projection of operator B, that meet positive and negative parts of the spectrum. It is not assumed that the operator B is reversible, in particular, B can have a nonzero kernel, and the spectrum of the operator B can contain both infinite subset of the positive and negative semi-axes.

Basic assumptions about the operators L, B are as follows.

I) L - maximal dissipative operator, and there is a Hilbert space F_1, densely imbedded in the E, such that $D(L^*) \subset F_1 \subset E$, and there is a constant $\delta_0 > 0$ such that $Re(-L^*u, u) \geq \delta_0\|u\|^2_{F_1}$ for all $u \in D(L^*)$, where L^* - adjoint operator.

The condition I) implies that the operator L^* – also maximal dissipative operator, and $0 \in \rho(L) \cap \rho(L^*)$, moreover, $\{Re\lambda \geq 0\} \subset \rho(L) \cap \rho(L^*)$.

II) The operator B is self-adjoint in E, and $F_1 \subset D(|B|^{1/2})$ densely.

Let F_2 – class of fuctions $u \in F_1$, such that u represented in the form $u = L^{-1}v$, where $v \in F_1'$, and $\|u\|_{F_2} = \|L^{-1}v\|_{F_1} + \|v\|_{F_1'} = \|u\|_{F_1} + \|Lu\|_{F_1'}$. Then $D(L) \subset F_2 \subset F_1$.

Define the space H as the completion of $D(|B|^{1/2})$ in the norm

$$\||B|^{1/2}v\| = \|u\|_H.$$

As follows from the definition, $|B|^{1/2} \in L(H, E)$.

Let $H_1 = \{v \in L_2(0, T; D(L^*)),\ v_t \in L_2(0, T; F_1)\}$.

Definition 1.1.1 *The function $u \in L_2(0, T; F_1)$ is called a generalized solution of problem (2), (3), if there are $\widetilde{u}_0, \widetilde{u}_T \in H$, such that $P^-\widetilde{u}_0 = \widetilde{u}_0, P^+\widetilde{u}_T = \widetilde{u}_T$, and the equality*

$$\int_0^T (-(Bu, v_t) - (u, L^*v))dt + (Bu_T, v(T)) - (Bu_0, v(0)) + \\ +(B\widetilde{u}_T, v(T)) - (B\widetilde{u}_0, v(0)) = \int_0^T (f, v)\, dt \tag{4}$$

for all $v \in L_2(0, T; D(L^*)), v_t \in L_2(0, T; F_1)$

Theorem 1.1.1 *Let that conditions I), II). Then for any $f \in L_2(0, T; F_1')$, $u_0, u_T \in H$ there exists a generalized solution $u \in L_2(0, T; F_1)$ of the boundary value problem (2), (3) in the sense of definition 1.1.1.*

Enter additional conditions:

III) $Re(-Lu, u) \geq \delta_0\|u\|^2_{F_1}, \quad \forall u \in D(L)$;

IV) There are constants $c > 0$ and $\theta \in (0, 1)$, such that

$$|(Bu, u)| \leq c\|u\|^{2\theta}_{F_1}\|L^{-1}Bu\|^{2(1-\theta)}_{F_1} \quad \forall u \in F_1;$$

V) $B|_{F_1} \in L(F_1, E)$.

Note that $\|L^{-1}Bu\|_{F_1} \leq c\|Bu\|_{F_1'} \leq c_1\|u\|_{F_1}$.

If the conditions of the Kato-sectorial operator L and the conditions of Theorem 1.1.1, we can show that the conditions III) and IV) are unnecessary, they are always satisfied.

Let $g(x)$ - almost everywhere positive in G function. We define the space $L_{2,g}(G; H)$ (H - Banach space) as the space of strongly measurable functions defined in G with values in H and such that

$$\|u\|_{L_{2,g}(G;H)} = \left(\int_G g(x)\|u(x)\|^2_H dx\right)^{1/2} < \infty.$$

Let $\varphi(t) = t^{2\alpha}(T - t)^{2\alpha}$, where $\alpha = \frac{1}{2(1-\theta)}$.

Theorem 1.1.2 *If the conditions I)-IV) and $f_t \in L_2(0, T; F_1')$, then the generalized solution obtained in Theorem 1.1.1, has the property that there is a generalized derivative $u_t \in L_{2,\varphi}(0, T; F_1)$. If in addition the condition V) and $f \in L_{2,\varphi}(0, T; E)$, then $u \in L_{2,\varphi}(0, T; D(L))$.*

In section 1.2 under the assumption that $\varphi_i(t) = t^{2i\alpha}(T-t)^{2i\alpha}$, where $\alpha = \frac{1}{2(1-\theta)}$, the following theorem is proved.

Theorem 1.2.1 *If the conditions I)-IV) and* $\partial_t^i f \in L_{2,\varphi_i}(0,T;F_1')$ $(i = 0,1,\ldots,m)$, *then generalized solution obtained in Theorem 1.1.1, has the property: There are generalized derivatives* ∂_t^i*and* $\in L_{2,\varphi_i}(0,T;F_1)$ $(i = 0,1,\ldots,m)$. *If in addition the condition V) and* $\partial_t^i f \in L_{2,\varphi_{i+1}}(0,T;E)$ $(i = 0,1,\ldots,m-1)$, *then* $u \in L_{2,\varphi_{i+1}}(0,T;D(L))$.

Section 1.3 is devoted to the study of boundary value problems for operator-differential equations of the form

$$B(t)u_t - L(t)u = f(t), \quad t \in (0,T), \quad T \leq \infty \tag{5}$$

where $L(t) : E \to E$ and $B(t) : E \to E$ – family of linear operators defined in a Hilbert space E. It does not assume that B is reversible.

It is further assumed that the operator $B(0) : E \to E$ and $B(T) : E \to E$ (if $T < \infty$) are self-adjoint. In this case, you can determine the spectral projections of $E^{\pm}(0)$, $E^{\pm}(T)$ of these operators, the respective positive and negative parts of the spectrum $B(0)$ and $B(T)$ respectively. For example, if $E_\lambda(0)$ – spectral decomposition of $B(0)$, then $E^-(0) = E_{-0}$, $E^+(0) = I - E_0$ (I – unit operator). Thus, $E_0^{\pm}B(0) = B(0)E_0^{\pm}$, $(E^+ - E^-)B(0) = |B(0)|$. We supplement the equation (5) boundary conditions

$$E^+(0)u(0) = u_0^+, \quad \lim_{t\to\infty} u(t) = 0 \quad (T = \infty), \tag{6}$$

$$E^+(0)u(0) = h_{11}E^-(0)u(0) + h_{12}E^+(T)u(T) + u_0^+, \tag{7}$$

$$E^-(T)u(T) = h_{21}E^-(0)u(0) + h_{22}E^+(T)u(T) + u_T^- \quad (T < \infty), \tag{8}$$

where h_{ij} are linear operators properties described later. The second condition in the (6) is the same as $u(t) \in L_2(0,\infty;E)$.

We fix a parameter $m = 0,1,2,\ldots$ and assume that the linear operators $L(t), B(t) : E \to E$, depending on the parameter $t \in (0,T)$, satisfy the following conditions.

(I) There is a complex Hilbert space H_1, densely stacked in the E, such that $L(t) \in L(t) \in W_\infty^m(0,T;L(H_1;H_1'))$ and $B(t) \in W_\infty^{\max(1,m)}(0,T;\, L(H_1;H_1'))$.

(II) Operators $B(t)$ $(t \in [0,T])$ are symmetric in the sense that $(B(t)u,v) = (u,B(t)v)$ for all $u,v \in H_1$. Operators $B(0) : E \to E$ and $B(T) : E \to E$ (if $T < \infty$) are self-adjoint in E; $H_1 \subset D(|B(0)|^{1/2})$ and $H_1 \subset D(|B(T)|^{1/2})$ and both inclusions are dense. Существует положительная постоянная $\delta_0 > 0$ такая, что

$$\operatorname{Re}((-L(t) + (i - \frac{1}{2})B_t(t))u,u) \geq \delta_0\|u\|_{H_1}^2, \; i = 0,1,2,\ldots,m$$

for all $u \in H_1$ and almost all $t \in (0,T)$.

Note that the $W_\infty^0(0,T;L(H_1;H_1')) = L_\infty(0,T;L(H_1;H_1'))$. When the condition (I) and change the operator-function $B(t)$ on a set of measure zero, if necessary, we can assume that

$B(t) \in C([0,T]; L(H_1; H_1'))$. Further we assume that this condition is satisfied. In addition, the condition (I) ensures that for every $u(t) \in W_2^1(0,T;H_1)$, the function $B(t)u(t) \in L_2(0,T;H_1')$ has generalized derivative $\frac{d}{dt}B(t)u(t)$, and

$$\frac{d}{dt}B(t)u(t) = B(t)u_t(t) + B_t(t)u(t),$$

where u_t, B_t – generalized derivatives of $u(t), B(t)$. Also $(B_t(t)u, v) = (u, B_t(t)v)$ for all $u, v \in H_1$ and almost all $t \in (0,T)$.

Define the space $F_0 = D(|B(0)|^{1/2})/\ker B(0)$, $G_0 = D(|B(T)|^{1/2})/\ker B(T)$, $F_0^{\pm} = \{u \in F_0 : E^{\pm}(0)u = u\}$, $G_0^{\pm} = \{u \in G_0 : E^{\pm}(T)u = u\}$. Let$F_1 = H_1/(\ker B(0) \cap H_1)$, $G_1 = H_1/(\ker B(T) \cap H_1)$. Condition (I) ensures that F_1, G_1 – dense subspace F_0, G_0 respectively. We introduce the norm in F_0, G_0 using equations $(u,v)_{F_0} = (B(0)J(0)u, v)$, $(u,v)_{G_0} = (B(T)J(T)u, v)$, where $J(0) = E^+(0) - E^-(0)$ and $J(T) = E^+(T) - E^-(T)$. Accordingly, the symbols $[u,v]_{F_0} = (B(0)u, v)$, $[u,v]_{G_0} = (B(T)u, v)$ denote the indefinite metric in F_0 and G_0. The norms and scalar products in $F_0^{\pm}$ and $G_0^{\pm}$ are similar to the norms and scalar products F_0, G_0 respectively. Construct space F_{-1} and G_{-1} as the completion of F_0, G_0 regulations regarding

$$\|u\|_{F_{-1}} = \|B(0)u\|_{H_1'}, \quad \|u\|_{G_{-1}} = \|B(T)u\|_{H_1'}.$$

We have $F_1 \subset F_0 \subset F_{-1}$ и $G_1 \subset G_0 \subset G_{-1}$.

Operators h_{ij} in (7), (8), are supposed to satisfy the conditions:

$$h_{11} \in L(F_0^-, F_0^+),\ h_{12} \in L(G_0^+, F_0^+),\ h_{21} \in L(F_0^-, G_0^-),\ h_{22} \in L(G_0^+, G_0^-). \tag{9}$$

We can define the operator

$$H = \begin{pmatrix} h_{11} & h_{1,2} \\ h_{21} & h_{22} \end{pmatrix} : F_0^- \times G_0^+ \to F_0^+ \times G_0^-.$$

In accordance with the terms on h_{ij} operator H defines a linear continuous mapping of $F_0^- \times G_0^+$ in $F_0^+ \times G_0^-$. We assume that its norm ρ_H satisfies

$$\rho_H < 1. \tag{10}$$

For $i \geq 1$ and $T < \infty$ let $\varphi_i(t) = t^{2i}(T-t)^{2i}$. For $T = \infty$, $\varphi_i \in C^\infty\big([0,\infty)\big)$, $\varphi_i(t) = t^{2i}$ for $t \leq 1$, $\varphi_i(t) = 2$ for $t \geq 2$, and $1 \leq \varphi_i(t) \leq 2$ for $t \in [1,2]$. Let $\varphi_0(t) \equiv 1$ and $\varphi_r = 1/\varphi_{-r}$ for $r < 0$. Given a Hilbert space H and integer $l \geq 0, s$, denoted by $W^{l,s}(H)$ $(l = 0, 1, \ldots)$ closure of $C_0^\infty(0,T;H)$ in the norm

$$\|v\|^2_{W^{l,s}(H)} = \sum_{i=0}^{l} \big\|\sqrt{\varphi_{i-s}}v^{(i)}\big\|^2_{L_2(0,T;H)}.$$

We have a $W^{0,0}(H) = L_2(0,T;H)$. The main results may be summarized as follows.

Theorem 1.3.1 *Suppose that* $T=\infty$, $f\in W^{m,0}(H_1')$ $(m=0,1,\ldots)$, $u_0^+\in F_0^+$, *conditions* (I)–(II) *and* (10) *are satisfied. Then there exists a solution* $u(t)\in W^{m,0}(H_1)$ *of the problem* (5), (6), *such that* $\sqrt{\varphi_i}\frac{d^{i+1}}{dt^{i+1}}B(t)u\in L_2(0,T;H_1')$, $i=0,1,\ldots,m$. *The trace* $u(0)\in F_{-1}$ *of this solution belongs to the space* F_0. *Equation* (5), *written in the form*

$$(Bu)_t-B_tu-Lu=f,$$

performed in the space $L_2(0,T;H_1')$. *If an additional* $m\geq 1$, $\sqrt{\varphi_1}f\in L_2(0,T;E)$, $B(t)\in L_\infty(0,T;L(H_1,E))$, *then we can assume that* $L(t)u(t)\in L_{2,\varphi_1}(0,T;E)$. *And if the operator* L *is independent of* t, $\sqrt{\varphi_{i+1}}\frac{d^i}{dt^i}f\in L_2(0,T;E)$ $(i=0,1,2,\ldots,m-1)$ *and* $B(t)\in W_\infty^{m-1}(0,T;L(H_1,E))$, *then* $\frac{d^i}{dt^i}Lu(t)\in L_{2,\varphi_{i+1}}(0,T;E)$ $(i=0,1,2,\ldots,m-1)$.

Theorem 1.3.2 *Suppose that* $T<\infty$, $f\in W^{m,0}(H_1')$ $(m=0,1,\ldots)$, $u_0^+\in F_0^+$, $u_T^-\in G_0^-$, *conditions* (I)–(II) *and* (10) *are satisfied. Then there exists a solution* $u(t)\in W^{m,0}(H_1)$ *of the problem* (5), (7), (8), *such that* $\sqrt{\varphi_i}\frac{d^{i+1}}{dt^{i+1}}B(t)u\in L_2(0,T;H_1')$, $i=0,1,\ldots,m$. *Traces* $u(0)\in F_{-1}$ *and* $u(T)\in G_{-1}$ (*для* $T\neq\infty$) *of these decisions belong to the spaces* F_0 *and* G_0 *respectively. Equation* (5), *written in the form*

$$(Bu)_t-B_tu-Lu=f,$$

performed in the space $L_2(0,T;H_1')$. *If further* $m\geq 1$, $\sqrt{\varphi_1}f\in L_2(0,T;E)$ *and* $B(t)\in L_\infty(0,T;L(H_1,E))$, *then we can assume that* $L(t)u(t)\in L_{2,\varphi_1}(0,T;E)$. *And if the operator* L *is independent of* t, $\sqrt{\varphi_{i+1}}\frac{d^i}{dt^i}f\in L_2(0,T;E)$ $(i=0,1,2,\ldots,m-1)$ *and* $B(t)\in W_\infty^{m-1}(0,T;L(H_1,E))$, *then* $\frac{d^i}{dt^i}Lu(t)\in L_{2,\varphi_{i+1}}(0,T;E)$ $(i=0,1,2,\ldots,m-1)$.

Lemma 1.3.3 *Let* $u(t)\in W$ *and*

$$(F_1,F_{-1})_{1/2,2}=F_0,\quad (G_1,G_{-1})_{1/2,2}=G_0. \tag{11}$$

Then traces $u(0)$ *and* $u(T)$ *belong to the spaces* F_0 *and* G_0 *respectively, and*

$$\|u(0)\|_{F_0}+\|u(T)\|_{G_0}\leq c\|u\|_W$$

for some number c, *does not depend on* u. *Moreover, all the elements* $v_0\in F_0$ *и* $v_T\in G_0$ *there is a function* $v(t)\in W$, *such that* $v(0)=v_0$ *и* $v(T)=v_T$.

Theorem 1.3.3 *Let condition* (11) *and the condition of the theorem 1.3.1 or Theorem 1.3.2 are fulfilled. Then there is no more than one solution of the problem* (5), (6) *of problem* (5), (7), (8) *respectively.*

The second chapter consists of three sections. In Section 2.1 presents the results of applications received in Chapter 1.

The equation of odd order with a changing direction of time

$$g(x)u_t-Lu=f, \tag{12}$$

$$u|_{t=0} = u_0(x),\, x \in G^+, \quad u|_{t=T} = u_T(x),\, x \in G^-, \tag{13}$$

$$u^{(i)}(0) = u^{(i)}(1) = 0,\, i = 0, 1, 2, ..., m-1, \tag{14}$$

$$u^{(m)}(0) = 0, \tag{15}$$

where L – differential operator of the form

$$Lu = (-1)^{m+1} \sum_{i=0}^{2m+1} a_i(x) u^{(i)}, \quad a_{2m+1} = 1, \tag{16}$$

$$L^* v = (-1)^{m+1} \sum_{i=0}^{2m+1} (a_i(x) v)^{(i)} \tag{17}$$

and $g(x) \in L_1(0,1)$ – real, measurable on $(0,1)$ function such that there exist open sets $G^+, G^- \subset (0,1)$ with the property $\mu(\overline{G}^+ \backslash G^+) = 0, \mu(\overline{G}^- \backslash G^-) = 0$, and $g(x) > 0$ almost everywhere on G^+, $g(x) < 0$ almost everywhere on G^- and $g(x) = 0$ on $G \backslash (\overline{G}^+ \cup \overline{G}^-)$.

We assume that

$$a_i \in W_\infty^i(0,1), \quad i = 0, 1, \ldots, 2m. \tag{18}$$

We believe that $D(L) = \{u \in W_2^{2m+1}(0,1) : u$ satisfies (14), (15)$\}$, $D(L^*) = \{u \in W_2^{2m+1}(0,1) :$ performed (14) and $u^{(m)}(1) = 0\}$ and there is a constant $\delta_0 > 0$ such that

$$Re(-Lu, u) \geq \delta_0 \|u\|^2_{W_2^m(0,1)},\, Re(-L^* u, u) \geq \delta_0 \|u\|^2_{W_2^m(0,1)} \tag{19}$$

for all $u \in D(L)$ and $u \in D(L^*)$ respectively.

Take

$$\theta = \frac{m+2-s}{2(m+1-s)}, \tag{20}$$

where $1/2 < s < m+1$, if $g \in L_1(0,1))$ and $0 \leq s < m+1, s \neq 1/2$, if $g \in L_2(0,1)$.

Theorem 2.1.1 *Suppose that the condition (19) and above conditions on the coefficients of the operator L (18) are satisfied and a function $g \in L_1(0,1)$. Then, if $f \in L_2(0,T; W_2^{-m}(0,1)), u_0 \in L_{2,g}(G^+), u_T \in L_{2,g}(G^-)$, there is a generalized solution of problem (12)-(15) of class $u \in L_2(0,T; W_2^m(0,1))$, $g(x)u_t \in L_2(0,T; (\widehat{W}_2^{m+1}(0,1))')$. If, in addition, suppose that $f_t \in L_{2,\varphi}(0,T; W_2^{-m}(0,1))$, where $\varphi(t) = t^{2\theta}(T-t)^{2\theta}$, and the parameter θ defined by the equation (20), then the solution has the following property: $u \in L_{2,\varphi}(0,T; W_2^{m+1}(0,1))$, $u_t \in L_{2,\varphi}(0,T; W_2^m(0,1))$. If in addition $g \in L_2(0,1)$, $f \in L_{2,\varphi}(0,T;\, L_2(0,1))$, hen the solution also has the property $u \in L_{2,\varphi}(0,T; W_2^{2m+1}(0,1))$. In the latter case, equation (12) holds almost everywhere in в $Q = (0,1) \times (0,T)$, and all generalized derivatives in equation exist.*

In Section 2.2 in field Q is considered a third-order equation changing time direction

$$\operatorname{sgn} x\, u_{ttt} + u_{xx} = f(x,t), \tag{21}$$

where Q is a rectangle $\Omega \times (0,T)$, $\Omega = (-1,1)$, $0 < T < +\infty$.

The solution $u(x,t)$ of the equation (21) is sought in the performance of initial conditions

$$\begin{aligned} &u(x,0) = u(x,T) = 0, \quad x \in \Omega, \\ &u_t(x,0) = u_0(x), \quad x \in (0,1), \quad u_t(x,T) = u_T(x), \quad x \in (-1,0) \end{aligned} \tag{22}$$

and homogeneous boundary conditions

$$u(-1,t) = u(1,t) = 0, \quad t \in (0,T). \tag{23}$$

We introduce the notation: $(u,v) = \int\limits_{\Omega} u\bar{v}\,dx$ — scalar product in $L_2(\Omega)$. By a generalized solution of problem (21)–(23) understand the function $u(x,t)$ such that $u \in \overset{\circ}{W}{}_2^1(Q)$, and performed the following integral identity

$$\begin{aligned} &\int\limits_0^T [(u_t,\, \operatorname{sgn} x\, v_{tt}) - (u_x,\, v_x)]\,dt + \int\limits_0^1 u_0(x) v_t(x,0)\,dx + \\ &+ \int\limits_{-1}^0 u_T(x) v_t(x,T)\,dx = \int\limits_0^T (f(x,t), v)\,dt \end{aligned} \tag{24}$$

for any function $v(x,t) \in \overset{\circ}{W}{}_2^1(Q)$, such that $v_{tt} \in L_2(Q)$, and satisfying the conditions

$$v_t(x,T) = 0, \quad 0 < x < 1, \quad v_t(x,0) = 0, \quad -1 < x < 0. \tag{25}$$

Denote by H_1 Hilbert space of functions $v(x,t) \in \overset{\circ}{W}{}_2^1(Q)$, such that $v_{tt} \in L_2(Q)$. As norm in H_1 we take value

$$\|u\|_{H_1} = (\|u\|^2_{\overset{\circ}{W}{}_2^1(Q)} + \|u_{tt}\|^2_{L_2(Q)})^{1/2}.$$

Theorem 2.2.1 *Let the function* $f(x,t) \in L_2(0,T; W_2^{-1}(\Omega))$, $u_0(x), u_T(x) \in L_2(\Omega)$. *Then the boundary value problem* (21) – (23) *has a generalized solution* $u(x,t) \in \overset{\circ}{W}{}_2^1(Q)$.

In section 2.3 in the field Q is considered a third-order equation changing time direction

$$\operatorname{sgn} x\, u_t - u_{xxx} = f(x,t). \tag{26}$$

The solution $u(x,t)$ of the equation (26) is sought in the performance of initial conditions

$$u(x,0) = u_0(x), \quad x \in (0,1), \quad u(x,T) = u_T(x), \quad x \in (-1,0) \tag{27}$$

and homogeneous boundary conditions

$$u(-1,t) = u_x(-1,t) = u(1,t) = 0, \quad t \in (0,T). \tag{28}$$

In the book of T.D. Dzuraev (1979) solvability of the boundary value problem for the equation (26) is reduced to the singular system of integral equations, which is in the class of regular solutions unambiguously and unconditionally solvable.

By a generalized solution of the boundary value problem (26) − (28) understand the function $u(x,t)$, such that $u \in L_2(0,T;\overset{\circ}{W}{}_2^1(-1,1)), u_t \in L_2(Q)$, and the following integral identity:

$$
\begin{gathered}
-\int_0^T [(u, \operatorname{sgn} x v_t) + (u_x, v_{xx})]\, dt = \\
= \int_0^T (f(x,t)v)\, dt + \int_{-1}^{0} u_T(x)\ v(x,T)dx + \int_0^1 u_0(x)v(x,0)dx
\end{gathered}
\tag{29}
$$

for any function $v(x,t) \in L_2(0,T;W_2^2(-1,1))$, such that $v_t \in L_2(Q)$, and satisfying the conditions

$$
\begin{gathered}
v(-1,t) = v(1,t) = 0, \quad v_x(1,t) = 0, \\
v(x,T) = 0, \quad 0 < x < 1, \qquad v(x,0) = 0, \quad -1 < x < 0.
\end{gathered}
\tag{30}
$$

Let H_2 the Hilbert space of functions $v(x,t) \in L_2(0,T;\ \overset{\circ}{W}{}_2^1(-1,1) \cap W_2^2(-1,1))$, such that $v_t \in L_2(Q)$ и $v_x(1,t) = 0$. As a norm in H_2 take the value

$$\|u\|_{H_2} = (\|u\|^2_{L_2(0,T;W_2^2(-1,1))} + \|u_t\|^2_{L_2(Q)})^{1/2}.$$

Theorem 2.3.1 *Let the function* $f(x,t) \in L_2(0,T;W_2^{-1}(\Omega))$, $u_0(x), u_T(x) \in L_2(\Omega)$. *Then the boundary value problem* (26) − (28) *has generalized solution* $u \in L_2(0,T;\overset{\circ}{W}{}_2^1(-1,1))$.

The third chapter examines the Gevrey third order equation with multiple features

$$\operatorname{sgn} x \cdot u_t - u_{xxx} = 0. \tag{31}$$

Part of the band Q, where $x < 0$ и $x > 0$, denoted by Q^- and Q^+. The solution of the equation is sought from holder spaces $H_{x\ t}^{p,p/3}(Q^\pm)$, $p = 3+\gamma$, $0 < \gamma < 1$, satisfying the following initial conditions

$$u(x,0) = \varphi_1(x),\ x > 0, \quad u(x,T) = \varphi_2(x),\ x < 0, \tag{32}$$

and bonding conditions

$$\frac{\partial^k u}{\partial x^k}(-0,t) = \frac{\partial^k u}{\partial x^k}(+0,t) \quad (k = 0,1,2). \tag{33}$$

The solvability of the boundary value problem (31)-(33) reduces to the solvability of the integral equation

$$\frac{4}{\sqrt{3}}\beta_1(t) + \frac{1}{\pi}\int_0^T \varphi\left(\frac{t}{\tau}\right)\frac{\beta_1(\tau)}{\tau}\, d\tau = Q_1(t), \quad \varphi(x) = x^{\frac{1-\gamma}{3}}\frac{1-x^{\frac{2}{3}}}{1-x}. \tag{34}$$

The integral equation (34) is an equation with a kernel that is homogeneous of degree −1. Introducing the new independent variables $t = Te^{-y}$, $\tau = Te^{-x}$, we have Wiener-Hopf integral equation

Theorem 3.2.1 *Let* $\varphi_1, \varphi_2 \in H^p$ $(p = 3+\gamma)$, $0 < \gamma < 1$. *Then under the 4 conditions of solvability* $L_s(\varphi_1,\varphi_2) = 0, \quad s = 1,2,3,4,$ *there exists a unique solution of the equation* (31) *at* Q *of space* $H_{x\ t}^{p,p/3}(Q^\pm)$, *satisfying the conditions* (32), (33).

Works to the theme of the dissertation

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RESEARCH OF THE SOLVABILITY OF BOUNDARY PROBLEMS FOR OPERATOR-DIFFERENTIAL EQUATIONS OF MIXED TYPE

ABSTRACT

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