Kuleshov Artur Vladimirovich

CONNECTIONS ON FAMILIES OF CENTERED PLANES IN PROJECTIVE SPACE

01.01.04 — geometry and topology

Abstract
of the PhD thesis

Kazan – 2016

The Thesis is done on the Chair of Fundamental Mathematics of
Immanuel Kant Baltic Federal University

Scientific Supervisor: **Shevchenko Yuri Ivanovich**, PhD,
assistant professor,
professor of the Chair of fundamental mathematics
of Immanuel Kant Baltic Federal University

Official Opponents: **Stepanov Sergey Evgenyevich**, PhD,
professor of the Chair "Mathematics"
of Financial University under
the Government of the Russian Federation
Sultanov Adgam Yachievich, PhD,
assistant professor, professor of the Chair "Geometry
and Analysis"
of Penza State University

Leading organization: Tver State University

The defence will take place in February 25, 2016 at 16 o'clock at the session of the dissertation council Д 212.081.10 under the Kazan (Volga Region) Federal University, address: 420008, Russia, Tatarstan Republic, Kazan city, Kremlyovskaya str, 35, room 610.

One can get familiar to the thesis in the Scientific Library of the Kazan (Volga Region) Federal University, address: 420008, Россия, Tatarstan Republic, Kazan city, Kremlyovskaya str, 18.

The abstract is sent «___» ________, 2016.

Scientific Secretary
of the Dissertation Council Д 212.081.10,
PhD, assistant professor E.K. Lipachev

GENERAL DESCRIPTION OF THESIS

Problem formulation and topicality of the subject. The theory of connections in principal bundles has an important role in the modern differential geometry. It has application in various branches of mathematics and physics, for example, in the field theory [4], [12]. Clothings are playing the main role in realization of connections on manifolds immersed in homogeneous of generalized spaces. The study of connections induced by various clothings of immersed manifolds had begun in the works of E. Cartan, E. Bortolotti, A.P. Norden, G.F. Laptev and others.

The first applications of the theory of connections to the projective differential geometry were given by E.Bortolotti [28] and E. Cartan [29]. E. Bortolotti had constructed the projective connection on equipped families of families of multi-dimensional planes. In his investigations the clothing of a family means attaching to any its m-dimensional plane L some $(n-m-1)$-dimensional plane $B_{n-m-1}(L)$ having no common points with L. Cartan proposed the analogous idea which is following. For any point of a surface S_m in the projective space P_n an $(n-m-1)$-dimensional plane $C_{n-m-1}(A)$, not intersecting the tangent plane $T_m(A)$ of the surface, is attached. Then on the equipped surface S_m the projective connection is appear. This connection is determined infinitesimally by projection with the center $C_{n-m-1}(A)$.

In general, a clothing of a submanifold is the attaching of each of its elements some additional geometric images in the ambient homogeneous space. Connections obtained is called a connection induced by clothing.

A.P. Norden [15] elaborated the normalization method which allowes one to induce two affine connections with vanishing torsion on a surface $S_m \subset P_n$. They are caled tangent and normal connections. The main idea of the method is attaching to any point $A \in S_m$ a pair of planes: 1) $(n-m)$-dimensional plane $N_{n-m}(A)$ passing through the point A and having no other than A common points with the tangent plane $T_m(A)$ of the surface S_m at A; 2) $(m-1)$-dimensional plane $N_{m-1}(A)$ lying in $T_m(A)$ and no passing through A. The plane $N_{n-m}(A)$ is called a first normal while $N_{m-1}(A)$ is called a second normal.

E.G. Neifeld in the paper [14] applied the Norden normalizaton method for inducing affine connection on the variety of all k-planes of n-dimensional projective space.

Simultaneously with development of general theory of immersed manifolds G.F. Laptev investigated the spaces with fundamental-group connection. Connections in the fiber bundles constructed by him either by means of mappings of infinitesimally closed fibers of the bundle [9], or by some object called the connection object [8]. Thus, the concept of connection arising in differential geometry as generalization of the notion of parallel displacement, later on became identified with a geometrical object of a certain type. Fruitfullness of this approach was demonstrated by L.E. Evtushik (see., e.g., [5]) while investigating nonlinear connections of the higher order.

Together with the theory of principal connections, the theory of connections in homogeneous bundles is constructed. Connecton in homogeneous bundle was defined by Yu.G. Lumiste [10], [11] as differentiable distribution satisfying some additional conditions.

Many concepts and claims about intrinsic geometry of immersed manifolds can be formulated in a natural way in terms of connections in fiber bundles associated with these manifolds. For example, the works of A.V. Chakmazyan [23], [24] cleared the meaning of connection in the normal subbundle.

The concept of r-dimensional family B_r of centered manifolds is one of the most interesting generalizations of surface, distribution of planes, hyperstrip and other geometric images on which the theory of connections is developing present time (see., e.g., [1], [2], [16], [20], [26], [27]). Due to the fact that every of these manifolds can be considered as a family of centered planes of a certain type, the investigation of generic family of centered planes has become an object of interest. It gives the following opportunities: 1) to construct a general theory of induced connections on the families of centered planes, which, in particular, will allow one to explain the cases of coinciding of the results obtained independently for various families; 2) to generalize the results obtained earlier for the concrete families to a more wide classes of the families of centered planes; 3) to obtain some new results which did not appeared in the works mentioned above.

The concept of family of centered planes relatively recently became the object of interest among geometers. This explains non-establishing terminology. So, together with the term “centered plane” which is used in the present work, the following synonyms, such as “planar element” “plane element”, “linear element” etc. can be found also (see., e.g., [9]). Centered plane is a non-complete flag in multidimensional projective space, and the studying of families of such an objects is included naturally in the theory of manifolds of figures and pairs of figures in a homogeneous space [13]. Besides, the concept of a family B_r is wider than the concept of a family of planes equipped by a field of points, and includes the latter as a particular case.

Among various mathematical constructions natural ones are distinguished. They do not contain any arbitrariness. Thus one of the main problems in the theory of connections is a problem of finding a natural construction for a connection on an immersed submanifold. The connection obtained such a way is called an intrinsical connection, or a connection constructed intrinsically. Such a connection is determined (induced) by the immersed manifold and does not require any additional structures. From the analytical point of view the problem of construction of intrinsic connection on an immersed manifold is reduced to the construction of some geometrical object scoped by the fundamental object of some order of the manifold.

When the structure of clothing induced a certain connection is found, the problem of construction of intrinsic connection reduces to the construction of intrinsic clothing. That is why the latter problem is one of the mailn problems of differential geometry of immerse manifolds [18]. For the families B_r this problem is far form its complete solution. Among numerous particular results we distinguish studying of intrinsic geometry of a regular hyperstrip [21], which can be included in the general theory of the families B_r as a special family of centered hyperplanes (of hyperplanar elements according to the terminilogy of [3]). Thus, the problem of extendind of constructions realized for hyperstrips on a maximally wide classes of the families of centered planes which are not investigated yet from the viewpoint of the problem of intrinsic clothing, is urgent.

The aim of thesis. The aim of the thesis is a construction of the general theory of fundamental-group connections on the families of centered planes in projective space.

The main tasks of the thesis:

1. To elaborate a universal method of inducing connections in principal bundles associated with an arbitrary family of centered planes;

2. To give geometric description of the induced connections, their bunches and bundles;

3. To investigate the curvature objects of the constructed connections;

4. To investigate connections on the families of the special type;

5. To construct intrinsic connections.

Methods of investigation. The results of the thesis are obtained using Cartan – Laptev method [6], [7], [19], [22].

Scientific novelty of results. All the results obtained in the thesis are new.

Statements for the thesis defence.

1. Three-parametric bundle $\Gamma(\xi,\, \eta,\, \zeta)$ of multiparametric bunches of fundamental-group connections has been constructed. Also one-parametric bundles of bunches of center-projective $\Gamma_1(\xi)$ and affine-group subconnections $\Gamma_2(\eta)$ on the family of centered planes have been constructed. From each bunch of this bundle one connection is distinguished. All together the distinguished connections form the three-parametric bundle.

2. Parallel displacements of clothing planes in the constructed bunches are described. Peculiarities of these displacements are revealed. These peculiarities are depend on the type of the bunch.

3. Geometric description of linear subconnections in terms of central projections is given.

4. It is shown that the curvature objects of fundamental-group connections of

various types are expressed in terms of the tensors of mobility. This allowes one to identify the dependence of vanishing these tensors and specialization of the clothing of the family.

5. The structure of a clothing of Grassman bundle on a surface in projective space inducing the bunch of associated fundamental-group connection is identified. It is shown that determining of an affine connection on the surface let one to distinguishing a connection from this bunch.

6. The problem of construction of intrinsic clothing and intrinsic connections on the family $\mathbb{B}$ of hyperplane elements is solved.

Theoretical and practical importance. The thesis has a theoretical importance. The obtained results and developed methods can be applied to studying connections on concrete figure families in homogeneous and generalized spaces. The theory developed in the thesis can be used as a special courses for magistrants and PhD students and for course papers, diploma works and magister works in I. Kant Baltic Federal University.

Approbation of thesis. The main results of the thesis have been presented and discussed on the seminars on geometry in I. Kant Baltic Federal University **1**. VII All-Russian youth scientific school-conference "Lobachevskie chteniya – 2008" (Kazan, December 1 – 3, 2008); **2**. The seminar on geometry in Kazan (Volga region) Federal university (Kazan, December 7, 2010); **3**. XLIX International scientific student conference "Student i nauchno-technichesky progress" (Novosibirskк, April 16 – 20, 2011); **4**. G.F. Laptev International geometry seminar "Laptevskie chteniya" (Penza, September 14 – 17, 2011); **5**. X All-Russian youth scientific school "Lobachevskie chteniya – 2011" (Kazan, October 31 – Novebmer 4, 2011); **6**. Seminar on the differential-geometric structure theory (Mocsow, December 12 – 16 , 2011, supervisor — professor, PhD L.E. Evtushik); **7**. Conference with international participation "Geometry of manifolds — 2012" (Ulan-Ude, June 20 – 23, 2012); **8**. International conference "AGMP–8" (Brno, Czech Republic, September 12 – 14, 2012), **9**. XI All-Russian youth scientific school-conference "Lobachevskie chteniya – 2012" (Kazan, November 1 – 6, 2012); **10**. XII All-Russian youth scientific school "Lobachevskie chteniya – 2013" (г. Казань, 24 – 29 октября, 2013 г.); **11**. G.F. Laptev International geometry seminar "Laptevskie chteniya – 2013" (г. Пенза, 11 – 15 сентября, 2013 г.); **12**. International conference "Geomerty days in Novosibirsk" (Novosibirsk, August 16 –19, 2015); **13**. G.F. Laptev International geometry seminar "Laptevskie chteniya – 2015" (Penza, September 9 – 12, 2015).

Publications. The mail scientific results included in the thesis were published in 19 publications. Their list is in the end of the abstract. It includes three papers published in journals form the Higher Attestation Comission list (the bold).

The author's contribution to the studying of the problems. The thesis is independent research of author. All the published papers are done without co-authors.

Structure and volume of the thesis. The thesis consists of the Introduction (historical overview, general description of the thesis, brief contents of the thesis), three Chapters and the Bibliography including 83 items. The whole volume of the thesis is 110 pages.

Author is sincerely grateful to his scienific supervisor Yu.I. Shevchenko for the proposed theme of research, attention to work and support.

THE MAIN CONTENTS OF THE THESIS

In the present thesis the families of centered planes in n-dimensional projective space are considered. All the functions and mappings introduced in the work are supposed to be smooth. All the considerations are local.

In the **Chapter 1** the generic family of centered planes B_r is studied. Here, B_r is considered as r-dimensional manifold of pairs $(A,\, L_m)$ in n-dimensional projective space P_n generated by a point A and a plane L_m passing through it, $dim L_m = m$, $1 \le r \le m(n-m)+n$.

§ 1.1 has a auxiliary aim. Here the structure equations of the projective space P_n $(I,\, J,\, K = \overline{1,n})$ are considered:

$$d\omega^I = \omega^J \wedge \omega^I_j, \quad d\omega_I = \omega^J_I \wedge \omega_J,$$
$$d\omega^I_J = \omega^K_J \wedge \omega^I_K + \delta^I_J \omega_K \wedge \omega^K + \omega_J \wedge \omega^I.$$

In § 1.2 the family B_r is given by the parametric equations expressing the principal forms ω^a, ω^α, ω^α_a of the family through the structure forms θ^i of the parameter space $(a,\, b,\, \ldots = \overline{1,\, m};\ \alpha,\, \beta,\, \ldots = \overline{m+1,\, n};\ i,\, j,\, \ldots = \overline{1,\, r})$:

$$\omega^a = \Lambda^a_i \theta^i, \quad \omega^\alpha = \Lambda^\alpha_i \theta^i, \quad \omega^\alpha_a = \Lambda^\alpha_{ai} \theta^i.$$

The differential equations on the components of the 1-st order fundamental object

$$\Lambda = \{\Lambda^a_i,\, \Lambda^\alpha_i,\, \Lambda^\alpha_{ai}\}$$

of the family B_r are obtained. The principal bundle $G_s(B_r)$ with the base B_r and the structure group G_s $(s = n(n+1) - m(n-m))$, which is the isotropy subgroup of the element $(A,\, L_m)$, is considered. The structure equations of this fiber bundle are found. It is shown that the fiber bundle has four quotient bundles: 1) the bundle of plane linear frames; 2) the bundle of normal linear frames; 3) the bundle of center-projective frames; 4) the bundle of affine-group frames.

The Laptev's fundamental-group connection of is given in the principal bundle in terms of the field if connection object by using the Laptev –Lumiste method (§ 1.3):

$$\Gamma = \{\Gamma^a_{bi},\, \Gamma^\alpha_{\beta i},\, \Gamma_{ai},\, \Gamma^a_{\alpha i},\, \Gamma_{\alpha i}\}.$$

It is shown that the object Γ includes two the simplest objects and two simple ones: 1) Γ^a_{bi} is the object of plane linear connection; 2) $\Gamma^\alpha_{\beta i}$ is the object of normal linear connection; 3) $\Gamma_1 = \{\Gamma^a_{bi},\, \Gamma_{ai}\}$ is the the object of center-projective connection; 4) $\Gamma_2 = \{\Gamma^a_{bi},\, \Gamma^\alpha_{\beta i},\, \Gamma^a_{\alpha i}\}$ is the object of affine-group connection. The object of curvature R of the connection Γ is a tensor consisting of two the simplest subtensors and two simple ones.

In § 1.4 the composite clothing of the family B_r is done. The clothing consists of the fields of analogues of the Norden second normals N_{m-1} and of analogues of the Cartan planes C_{n-m-1}. Form the analytical point of view the clothing is determined by the field of the quasitensor $\lambda = \{\lambda_a, \lambda^a_\alpha, \lambda_\alpha\}$ (so-called clothing quasitensor). the object λ' consisting of the pfaffian derivatives of the components of the quasitensor λ forms a geometrical object only together with the objects λ and Λ. The plane $N_{n-m} = C_{n-m-1} \oplus A$ is an analogue of the Norden first normal generated by the Cartan plane. The plane $P_{n-1} = N_{m-1} \oplus C_{n-m-1}$ is an analogue of the Bortolotti hyperplane and spans the Cartan plane and the Norden second normal. The differentials of the point A and of the base points of the clothing planes are decomposed on the same points. the coefficients in these decompositions Λ^α_i, M^a_i, M^α_{ai}, t_{ai}, $t^a_{\alpha i}$, $t_{\alpha i}$ form tensors.

Vanishing of the subtensor Λ^α_i of the fundamental tensor Λ distinguishes the families B_r such that any center A of L^*_m moves inside this plane. Vanishing of the tensor M^a_i distinguishes equipped families B_r such that centers of its planes L^*_m move inside the corresponding first normals N_{n-m}. Vanishing of the tensor M^α_{ai} distinguish the families such that the normals N_{m-1} are moving inside the corresponding planes L^*_m. The cases of vanishing of the tensors t_{ai}, $t^a_{\alpha i}$, $t_{\alpha i}$ are described geometrically by the special motions of the clothing planes, in these cases no restriction is imposed on the very family B_r. That is why we call the tensors t_{ai}, $t^a_{\alpha i}$, $t_{\alpha i}$ tensors of mobility of the clothing planes.

In § 1.5 covariant derivatives $\nabla_i\lambda_a$, $\nabla_i\lambda^a_\alpha$, $\nabla_i\lambda_\alpha$ of the components of the clothing quasitensor λ with respect to the connection Γ are considered. The tensor

$$T_{\alpha i} = \nabla_i\lambda_\alpha - \lambda_a\nabla_i\lambda^a_\alpha$$

is constructed also. An invariant relations between them and the mobility tensors

$$\nabla_i\lambda_a = \xi t_{ai}, \quad \nabla_i\lambda^a_\alpha = \eta t^a_{\alpha i}, \quad T_{\alpha i} = \zeta t_{\alpha i},$$

containing the numerical parameters ξ, η, ζ, distinguish the three-parametric bundle of bunches of the fundamental-group connections

$$\Gamma(\xi, \eta, \zeta) = \{\Gamma^a_{bi}, \Gamma^\alpha_{\beta i}, \Gamma_{ai}(\xi), \Gamma^a_{\alpha i}(\eta), \Gamma_{\alpha i}(\xi, \eta, \zeta)\}.$$

This bundle contains the center-projective $\Gamma_1(\xi) = \{\Gamma^a_{bi}, \Gamma_{ai}(\xi)\}$ and the affine-group $\Gamma_2(\eta) = \{\Gamma^a_{bi}, \Gamma^\alpha_{\beta i}, \Gamma^a_{\alpha i}(\eta)\}$ subbundles.

Using the scopes of the components Γ^a_{bi} and $\Gamma^\alpha_{\beta i}$ the threee-parametric bundle of induced connections $\overset{0}{\Gamma}(\xi, \eta, \zeta)$ is distinguished form the bundle $\Gamma(\xi, \eta, \zeta)$ (§ 1.5). It is shown that the coincidence of the constructed connections corresponds to a specialization of composite clothing of the family B_r.

In § 1.6 a geometrical description of the induced plane $\overset{0}{\Gamma}{}^a_{bi}$ and normal linear connections $\overset{0}{\Gamma}{}^\alpha_{\beta i}$ is given. The connection $\overset{0}{\Gamma}{}^a_{bi}$ is described geometrically by the projection on the second normal N_{m-1} of the near normal $N_{m-1} + dN_{m-1}$ from the center N_{n-m} of the projection:

$$\overset{0}{\Gamma}{}^a_{bi}\colon\ N_{m-1} + dN_{m-1} \overset{N_{n-m}}{\longrightarrow} N_{m-1}.$$

The connection $\overset{0}{\Gamma}{}^\alpha_{\beta i}$ is described geometrically by the projection on the Cartan plane C_{n-m-1} of the near plane $C_{n-m-1} + dC_{n-m-1}$ from the center L_m of the projection:

$$\overset{0}{\Gamma}{}^\alpha_{\beta i}\colon\ C_{n-m-1} + dC_{n-m-1} \overset{L_m}{\longrightarrow} C_{n-m-1}.$$

The sections §§ 1.7 – 1.12 are about an interpretation of the connections constructed and their center-projective and affine-group subconnections. For this goal non-degenerated, free-degenerated connected-degenerated parallel displacements of the clothing planes in the corresponding bunches of connections. The existence conditions of the parallel displacements introduced are some restrictions of the mobility tensor matrix ranks. Using these conditions the arbitrariness of any concrete parallel displacement is obtained (the dimension of so-called parallelism subspace).

In § 1.13 the expressions of the components of the curvature tensors of the induced connections of the bundle $\overset{0}{\Gamma}(\xi,\,\eta,\,\zeta)$:

$$\overset{0}{R}{}^a_{bij} = M^\alpha_{b[i}t^a_{\alpha j]} - \delta^a_b\Lambda^\alpha_{[i}t_{\alpha j]} - M^c_{[i}(\delta^a_c t_{bj]} + \delta^a_b t_{cj]}),$$

$$\overset{0}{R}{}^\alpha_{\beta ij} = -M^\alpha_{a[i}t^a_{\beta j]} - \Lambda^\alpha_{[i}t_{\beta j]} - \delta^\alpha_\beta(M^a_{[i}t_{aj]} + \Lambda^\gamma_{[i}t_{\gamma j]}),$$

$$\overset{0}{R}_{aij}\ (\xi) = \overset{0}{R}{}^b_{aij}\ \lambda_b + \xi M^\alpha_{a[i}t_{\alpha j]},$$

$$\overset{0}{R}{}^a_{\alpha ij}\ (\eta) = \overset{0}{R}{}^\beta_{\alpha ij}\ \lambda^a_\beta - \overset{0}{R}{}^a_{bij}\ \lambda^b_\alpha + \eta M^a_{[i}t_{\alpha j]},$$

$$\overset{0}{R}_{\alpha ij}\ (\xi,\,\eta,\,\zeta) = \overset{0}{R}{}^\beta_{\alpha ij}\ \lambda_\beta - \overset{0}{R}{}^b_{aij}\ \lambda^a_\alpha\lambda_b - \xi M^\beta_{a[i}t_{\beta j]}\lambda^a_\alpha + \\ + (\zeta - \xi\eta)t^a_{\alpha[i}t_{aj]} + \eta\lambda_a M^a_{[i}t_{\alpha j]}.$$

The conditions of their coincidence for various induced center-projective, affine-group and fundamental-group connections are obtained.

In § 1.14 the flat connections (i.e. with the vanishing curvature tensor) are considered. Five sufficient conditions for the flatness of any connection from the bundle $\overset{0}{\Gamma}(\xi,\,\eta,\,\zeta)$ are obtained.

In the Chapter 2 the Grassman bundle BS of centered planes is considered. BS is the union of centered Grassman manifolds $Gr^*(m,\,n)$ in all tangent planes of some n-dimensional surface S_n in N-dimensional projective space P_N. The section § 2.1 is expository, there some basic information about surface in multidimensional projective space is provided. In § 2.1 various clothings of the surface are considered such as the normalization of the first and the second kinds and the subnormalization of the first kind. It is shown that the first kind normalization induces the first kind subnormalization, while the latter together with the second kind normalization induce the affine connection Γ^I_{JK} $(I,\, J,\, \ldots = \overline{1,\, n})$ in the linear frame bundle $L(S_n)$.

In § 2.3 the family BS of m-dimensional centered planes is considered. Let H be the isotropy subgroup of the element of the family BS inside the corresponding tangent plane of the surface. The principal bundle $H(BS)$ with the base BS and the structure group H is considered. In $H(BS)$ the connection Π is given and the equations on the object Π are obtained (§ 2.4). The section § 2.5 is about clothings of the manifold BS. The composite clothing of the family BS induced by the first normalization of BS together with the second normalization of the surface S_n is called an N-induced clothing. In § 2.6 it is shown that the composite clothing of the manifold BS reduces the fundamental-group connection Π to the plane linear subconnection Π_1 and the normal linear one Π_2. In other words, it induces in $H(BS)$ multi-parametric bunch of the fundamental-group connections Π.

the first normalization of BS together with the affine connection Γ^I_{JK} on S_n induce the plane linear connection Π_1 and the normal linear one Π_2. This allows one to distinguish a fundamental-group connection $\overset{0}{\Pi}$ form the constructed bunch.

In the **Chapter 3** the family $\mathbb{B}$ of hyperplane elements (i.e. centered hyperplanes) in multidimensional projective space is considered. In § 3.1 necessary definitions are given and the problem of construction of composite clothing of this family intrinsically is posed. The solution of this problem is based on reduction of the frame bundle adapted to this family (§ 3.2). In the sections §§ 3.2 – 3.5 the stepwise description of this procedure is provided. This procedure includes the analytical canonisation of a frame based on applying the Ostianu lemma [17] systematically.

In § 3.6 two applications of the obtained results are mentioned. Firstly, in the special case the canonical structure of quasi-product on the family $\mathbb{B}$ has been found. Secondly, in the generic case two linear connections are attached to the family $\mathbb{B}$ and the expressions on their curvature tensors are obtained.

BIBLIOGRAPHY

1. Belova, O. O. Connections in fiberings associated with the Grassman manifold and the space of centered planes / O.O. Belova // Fundamental and Applied mathematics [in Russian]. — 2008. — Vol. 14. — №2. — pp. 29 – 67.

2. Bondarenko, E. V. Connections on a manifold of centered planes in a projective space / E.V. Bondarenko // Differential Geometry of Figure Manifolds. — Kaliningrad, 2000. — No. 31. — pp. 12 – 16.

3. Bochillo, G. P. On the differential geometry of m-distributions on the manifold of all the hyperplane elements of the n-dimensional projective space / G.P. Bochillo // Differential Geometry of Figure Manifolds. — Kaliningrad, 1983. — No. 14. — pp. 18 – 23.

4. Volobuev, I. P. Differential geometry and Lie algebras and their appllications to the field theory / I.P. Volobuev, Yu.A. Kubyshin. — 3-rd ed. — M.: LENAND, 2015. — 232 p.

5. Evtushik, L. E. Nonlinear connections of the higher orders / L.E. Evtushik // Izvestiya VUZ. Mathematika. — 1969. — №2. — pp. 32 – 44.

6. Evtushik, L. E. Differential-geometric structures on manifolds / L. E. Evtushik, Yu. G. Lumiste, N.M. Ostianu, A.P. Shirokov // Problems of Geometry. . — M.: VINITI, 1979. — Vol. 9. — pp. 5 – 247.

7. Laptev, G. F. Differential geometry of immersed manifolds / G.F. Laptev // Tr. Mosk. Mat. Obshch. — M., 1953. — Vol. 2. — pp. 275 – 382.

8. Laptev, G. F. The main infinitesimal structures of the higher order s on a smooth manifold / G.F. Laptev // Proc. of Geom. Semin., — M.: VINITI, 1966. — Vol. 1. — pp. 139 – 189.

9. Laptev, G. F. Distributions of m-dinensional linear elements in the space of projective connection. I / G.F. Laptev, N.M. Ostianu // Proc. of Geom. Semin. — M.: VINITI, 1971. — Vol. 3. — pp. 49 – 83.

10. Lumiste, Yu.G. Homogeneous fiber bundles with a connection and their immersions / Yu.G. Lumiste // Proc. of Geom. Semin. — M.: VINITI, 1966. — Vol. 1. — pp. 191 – 237.

11. Lumiste, Yu.G. Connections in homogeneous bundles / Yu.G. Lumiste // Math. Sbornik. — 1966. — Vol. 69. — pp. 434 – 469.

12. Lumiste, Yu.G. Connections in geometric interpretation of Jang – Mills fields and Faddeev – Popov / Yu.G. Lumiste // Izvestiya VUZ. Mathematika. — 1983, — № 1. — pp. 46 – 54.

13. Malakhovsky, V.S. Differential gometry of manifolds of figures and of pairs of figures in a homogeneous space / V.S. Malakhovsky // Proc. of Geom. Semin. — M.: VINITI, 1969. — Vol. 2. — pp. 179 – 205.

14. Neifeld, E.G. Affine connections in normalized manifold of planes in projective space / E.G. Neifeld // Izvestiya VUZ. Mathematika. — 1976. — № 11. — pp. 48 – 55.

15. Norden, A.PП. Spaces of affine connection / A.P. Norden. — M.: Nauka, 1976. — 432 p.

16. Omelyan, O.M. Reductions of the object of center-projective connection and the affine torsion tensor on a distribution of planes / O.M. Omelyan, Yu.I. Shevchenko // Math. Zametki. — M., 2008. — Vol. 84. — No. 1. — pp. 99 – 107.

17. Ostianu, N.M. On canonization of moving frame of an immersed manifold / N.M. Ostianu // Rev. math. pures et appl. (RPR). — 1962. — Vol. 7. — № 2. — pp. 231 – 240.

18. Ostianu, N.M. On the invariant clothing of a family of multidimensional planes in a projective space / N.M. Ostianu // Proc. of Geom. Semin. — M.: VINITI, 1969. — Vol. 2. — pp. 247 – 262.

19. Ostianu, N.M. The Cartan –Laptev method in studying G-structures on manifolds / N.M. Ostianu // Itogi Nauki i techniki. Sovrem. Math. i prilozh. — M.: VINITI, 2002. — Vol. 30. — pp. 5 – 124.

20. Polyakova, K.V. Parallel displacements on the surface of a projective space / K.V. Polyakova // Fundamental and Applied mathematics [in Russian]. — 2008. — Vol. 14. — № 2. — pp. 129 – 177.

21. Stolyarov, A.V. On fundamental objects odf a regular hyperstrip / A.V. Stolyarov // Izvestiya VUZ. Mathematika. — 1975. — № 10. — pp. 97 – 99.

22. Finikov, S.P. The Cartan method of exterior forms in differential geometry. The theory of compatibility of systems of differential equations and of partial differential equations / S.P. Finikov. — Moscow.-Leningrad.: GITTL, 1948. — 432 pp.

23. Chakmazyan, A.V. On the normal connection of a normalized manifolds of planes in a projective space / A.V. Chakmazyan // Izvestiya VUZ. Mathematika. — 1974. — № 7. — pp. 74 – 79.

24. Chakmazyan, A.V. Connection in normal bundles of the normalized submanifold V_m in P_n / A.V. Chakmazyan // Problems of Geometry. — Vol. 10. — M.: VINITI, 1978. — pp. 55 – 74.

25. Shevchenko, Yu.I. On clothings of manifolds of planes in a projective space / Yu.I. Shevchenko // Differential Geometry of Figure manifolds. — Kaliningrad, 1978. — Vol. 9. — pp. 124 – 133.

26. Shevchenko, Yu.I. Geometric description of some induced connections of a surface / Yu.I. Shevchenko // Differential Geometry of Figure manifolds. — Kaliningrad, 1981. — Vol. 12. — pp. 126 – 130.

27. Shevchenko, Yu.I. Parallel displacement of a figure in a linear combination of connections / Yu.I. Shevchenko // Differential Geometry of Figure manifolds. — Kaliningrad, 1987. — Vol. 18. — pp. 115 – 120.

28. Bortolotti, E. Connessioni nelle varieta luogo di spazi / E. Bortolotti // Rend. Semin. Fac. Sci. Univ. Cagliari. — 1933. — № 3. — P. 81 – 89.

29. Cartan, E. Leçons sur la théorie des espaces à connexion projective / E. Cartan. — Paris: Gauthier-Villars, 1937. — 308 p.

ПУБЛИКАЦИИ АВТОРА ПО ТЕМЕ ДИССЕРТАЦИИ

1. Kuleshov, A. V. On connections of the 1-st and the 2-nd orders on a manifold of centered planes in a projective space / А. В. Кулешов // Proc. of Lobachevsky Math. Center. — Kazan: Kazan math. Soc., 2007. — Vol 36. — pp. 131 – 133. — 0,19 pr.p.

2. Kuleshov, A. V. Induced connections on a manifold of centered planes / A. V. kuleshov // Proc. of XX International summer school-seminar on the modern problems of Theoretical and Math. Physics. "Volga'20 – 2008". — Kazan, 2008. — p. 38. — 0,06 pr.p.

3. Kuleshov, A. V. On curvature tensors of induced group connection on a manifold of centered planes / A. V. Kuleshov // Proc. of Lobachevsky Math. Center. — Kazan: Kazan math. Soc., 2008. — Vol. 37. — pp. 133 – 135. — 0,19 pr.p.

4. Kuleshov, A. V. The Cartan – Norden clothing of a congruence of centered planes / A. V. Kuleshov // Proc. of Internatonal Conf. "Geometry in Odessa – 2009". — Odessa: "Nauka", 2009. — p. 54. — 0,06 pr.p.

5. Kuleshov, A. V. On connections on a congruence of centered planes / A. V. Kuleshov // Proc. of XXI International Summer school-seminar on the modern problems of Theoretical and Math. Physics. "Volga'20 – 2009". — Kazan, 2009. — p 26. — 0,06 pr.p.

6. Kuleshov, A. V. Six types of induced group connections on a family of centered planes / A. V. Kuleshov // Differential Geometry of Figure manifolds. — Kaliningrad, 2009. — No. 40. — pp. 72 – 84. — 0,81 pr.p.

7. Kuleshov, A. V. On coincidence and interpretation of connections induced on a family of centered planes / A. V. Kuleshov // Vestnik Ross. Gos. Univ. I. Kanta. Ser. Phys.-Math.Sciences. — Kaliningrad, 2009. — Vol. 10. — pp. 112 – 119. — 0,50 pr.p.

8. Kuleshov, A. V. Generalized connections on a complex of centered planes in a projective space / A. V. Kuleshov // Proc. of intern. Conf. "Petrov 2010 Anniversary Symposium on General Relativity and Gravitation". — Kazan, 2010. — p. 80. — 0,06 pr.p.

9. Kuleshov, A. V. Connections induced by the composite clothing of the family of centered planes in projective space / A. V. Kuleshov // Proc. of XLIX Intern. Scientific Student Conf. "Student and scientific-techncal progress": Mathematics. — Novosibirsk, 2011. — p. 79. — 0,06 pr.p.

10. Kuleshov, A. V. Connections induced by the composite clothing of the family of centered planes in a projective space / A. V. Kuleshov // Differential Geometry of Figure manifolds. — Kaliningrad, 2011. — Vol. 42. — pp. 59 – 87. — 1,81 pr.p.

11. Kuleshov, A. V. Three-parametrical bundle of induced fundamental-group connections on a family of centered planes in a projective space / А. В. Кулешов // Proc. of Lobachevsky Math.Center. — Kazan: Kazan Math. Society, 2011. — Vol. 44. — pp. 189 – 192. — 0,25 pr.p.

12. **Kuleshov, A. V. Curvature of the induced fundamental-group connections of a family of centered planes in a projective space / A. V. Kuleshov // Izvestiya of Belinsky Penza State Pedagogical University. — Penza, 2011. — №, 26. – pp. 111 – 120. — 0,63 pr.p.**

13. Kuleshov, A. V. Special clothings inducing flat connections on a surface in projective space / A. V. Kuleshov // Proc. of Intern. Conf. "Geometry in Odessa – 2012". — Odessa: "Nauka", — 2012. — p. 51. — 0,06 pr.p.

14. **Kuleshov, A. V. Fundamental-group connections induced by the composite clothing of the family of centered planes in projective space / A. V. Kuleshov // Vestnik of I. Kant Baltic Federal university. Ser. Phys.-Math. sciences. — Kaliningrad, 2012. — Vol. 4. — pp. 139 – 147. — 0,56 pr.p.**

15. **Kuleshov, A. V. Induced connection on the Grassman bundle ovar a manifold in a projective space / A. V. Kuleshov // Izvestiya of Belinsky Penza State Pedagogical University. — № 30. — Penza, 2012. — pp. 84 – 88. — 0,31 п.л.**

16. Kuleshov, A. V. On some projective invariant of a family of hyperplane elements with enveloping surface of centers / A. V. Kuleshov // Proc. of Intern. Conf. “Geometry in Odessa — 2013”. — Odessa: “Nauka”, 2013. — p. 53. — 0,06 pr.p.

17. Kuleshov, A. V. On some projective invariant of a family of hyperplane elements with enveloping surface of centers / A. V. Kuleshov // Differential Geometry of Figure manifolds. — Kaliningrad, 2013. —No. 44. — pp. 67 – 77. — 0,69 pr.p.

18. Kuleshov A. V. Intrinsic clothing of regular family of hyperplane elements / A. V. Kuleshov // Proc. of Intern. Conf. “Geometry in Odessa — 2014”. — Odessa: “Nauka”, 2014. — C. 78. — 0,06 pr.p.

19. Kuleshov, A. V. On intrinsic clothing of some family of hyperplane elements / A. V. Kuleshov // Differential Geometry of Figure manifolds. — Kalininngrad, 2014. —No. 45. — pp. 65 – 72. — 0,50 pr.p.