Eurasian Mathematical Journal 2021 vol.12 N3, pages 46-56

SHARP CONFORMALLY INVARIANT HARDY-TYPE INEQUALITIES WITH REMAINDERS

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Abstract

In the present paper we establish new Hardy-Maz'ya-type inequalities with remainders for all continuously differentiable functions with compact support in the half space R n+. The weight functions depend on the distance to the boundary or on the distance to the origin. Also new sharp Avkhadiev-Hardy-type inequalities involving the distance to the boundary or the hyperbolic radius are proved. We consider Avkhadiev-Hardy-type inequalities in simply and doubly connected plain domains and in tube-domains

http://dx.doi.org/10.32523/2077-9879-2021-12-3-46-56

Keywords

Distance function, Half space, Hardy inequality, Hyperbolic domain, Hyperbolic radius, Remainder terms, The poincare metric

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