

# Measurement of the casimir force between 0.2 and 8 $\mu\text{m}$ : Experimental procedures and comparison with theory

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## Abstract

We present results on the determination of the differential Casimir force between an Au-coated sapphire sphere and the top and bottom of Au-coated deep silicon trenches performed by means of the micromechanical torsional oscillator in the range of separations from 0.2 to 8  $\mu\text{m}$ . The random and systematic errors in the measured force signal are determined at the 95% confidence level and combined into the total experimental error. The role of surface roughness and edge effects is investigated and shown to be negligibly small. The distribution of patch potentials is characterized by Kelvin probe microscopy, yielding an estimate of the typical size of patches, the respective r.m.s. voltage and their impact on the measured force. A comparison between the experimental results and theory is performed with no fitting parameters. For this purpose, the Casimir force in the sphere-plate geometry is computed independently on the basis of first principles of quantum electrodynamics using the scattering theory and the gradient expansion. In doing so, the frequency-dependent dielectric permittivity of Au is found from the optical data extrapolated to zero frequency by means of the plasma and Drude models. It is shown that the measurement results exclude the Drude model extrapolation over the region of separations from 0.2 to 4.8  $\mu\text{m}$ , whereas the alternative extrapolation by means of the plasma model is experimentally consistent over the entire measurement range. A discussion of the obtained results is provided.

<http://dx.doi.org/10.3390/universe7040093>

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## Keywords

Casimir force, Comparison between experiment and theory, Drude model, Gradient expansion, Micromechanical torsional oscillator, Plasma model, Precise measurements, Scattering theory

## References

- [1] Casimir, H.B.G. On the attraction between two perfectly conducting plates. Proc. K. Ned. Akad. Wet. B 1948, 51, 793–795.
- [2] Lamoreaux, S.K. Demonstration of the Casimir Force in the 0.6 to 6  $\mu\text{m}$  Range. Phys. Rev. Lett. 1997, 78, 5–8. [CrossRef]
- [3] Lifshitz, E.M. The theory of molecular attractive forces between solids. Zh. Eksp. Teor. Fiz. 1955, 29, 94–110; Translated: Sov. Phys. JETP 1956, 2, 73–83.
- [4] Dzyaloshinskii, I.E.; Lifshitz, E.M.; Pitaevskii, L.P. The general theory of van der Waals forces. Usp. Fiz. Nauk 1961, 73, 381–422; Translated: Adv. Phys. 1961, 10, 165–209. [CrossRef]

- [5] Mohideen, U.; Roy, A. Precision Measurement of the Casimir Force from 0.1 to 0.9  $\mu\text{m}$ . *Phys. Rev. Lett.* 1998, 81, 4549-4552. [CrossRef]
- [6] Chan, H.B.; Aksyuk, V.A.; Kleiman, R.N.; Bishop, D.J.; Capasso, F. Quantum mechanical actuation of microelectromechanical system by the Casimir effect. *Science* 2001, 291, 1941-1944. [CrossRef] [PubMed]
- [7] Chan, H.B.; Aksyuk, V.A.; Kleiman, R.N.; Bishop, D.J.; Capasso, F. Nonlinear Micromechanical Casimir Oscillator. *Phys. Rev. Lett.* 2001, 87, 211801. [CrossRef]
- [8] Decca, R.S.; López, D.; Fischbach, E.; Krause, D.E. Measurement of the Casimir Force between Dissimilar Metals. *Phys. Rev. Lett.* 2003, 91, 050402. [CrossRef] [PubMed]
- [9] Decca, R.S.; Fischbach, E.; Klimchitskaya, G.L.; Krause, D.E.; López, D.; Mostepanenko, V.M. Improved tests of extra-dimensional physics and thermal quantum field theory from new Casimir force measurements. *Phys. Rev. D* 2003, 68, 116003. [CrossRef]
- [10] Decca, R.S.; López, D.; Fischbach, E.; Klimchitskaya, G.L.; Krause, D.E.; Mostepanenko, V.M. Precise comparison of theory and new experiment for the Casimir force leads to stronger constraints on thermal quantum effects and long-range interactions. *Ann. Phys. (NY)* 2005, 318, 37-80. [CrossRef]
- [11] Decca, R.S.; López, D.; Fischbach, E.; Klimchitskaya, G.L.; Krause, D.E.; Mostepanenko, V.M. Tests of new physics from precise measurements of the Casimir pressure between two gold-coated plates. *Phys. Rev. D* 2007, 75, 077101. [CrossRef]
- [12] Decca, R.S.; López, D.; Fischbach, E.; Klimchitskaya, G.L.; Krause, D.E.; Mostepanenko, V.M. Novel constraints on light elementary particles and extra-dimensional physics from the Casimir effect. *Eur. Phys. J. C* 2007, 51, 963-975. [CrossRef]
- [13] Palik, E.D. (Ed.) *Handbook of Optical Constants of Solids*; Academic Press: New York, NY, USA, 1985.
- [14] Chang, C.C.; Banishev, A.A.; Castillo-Garza, R.; Klimchitskaya, G.L.; Mostepanenko, V.M.; Mohideen, U. Gradient of the Casimir force between Au surfaces of a sphere and a plate measured using an atomic force microscope in a frequency-shift technique. *Phys. Rev. B* 2012, 85, 165443. [CrossRef]
- [15] Banishev, A.A.; Klimchitskaya, G.L.; Mostepanenko, V.M.; Mohideen, U. Demonstration of the Casimir Force between Ferromagnetic Surfaces of a Ni-Coated Sphere and a Ni-Coated Plate. *Phys. Rev. Lett.* 2013, 110, 137401. [CrossRef] [PubMed]
- [16] Banishev, A.A.; Klimchitskaya, G.L.; Mostepanenko, V.M.; Mohideen, U. Casimir interaction between two magnetic metals in comparison with nonmagnetic test bodies. *Phys. Rev. B* 2013, 88, 155410. [CrossRef]
- [17] Bimonte, G. Hide It to See It Better: A Robust Setup to Probe the Thermal Casimir Effect. *Phys. Rev. Lett.* 2014, 112, 240401. [CrossRef] [PubMed]
- [18] Bimonte, G.; López, D.; Decca, R.S. Isoelectronic determination of the thermal Casimir force. *Phys. Rev. B* 2016, 93, 184434. [CrossRef]
- [19] Xu, J.; Klimchitskaya, G.L.; Mostepanenko, V.M.; Mohideen, U. Reducing detrimental electrostatic effects in Casimir-force measurements and Casimir-force-based microdevices. *Phys. Rev. A* 2018, 97, 032501. [CrossRef]
- [20] Liu, M.; Xu, J.; Klimchitskaya, G.L.; Mostepanenko, V.M.; Mohideen, U. Examining the Casimir puzzle with an upgraded AFM-based technique and advanced surface cleaning. *Phys. Rev. B* 2019, 100, 081406(R). [CrossRef]
- [21] Liu, M.; Xu, J.; Klimchitskaya, G.L.; Mostepanenko, V.M.; Mohideen, U. Precision measurements of the gradient of the Casimir force between ultraclean metallic surfaces at larger separations. *Phys. Rev. A* 2019, 100, 052511. [CrossRef]
- [22] Sushkov, A.O.; Kim, W.J.; Dalvit, D.A.R.; Lamoreaux, S.K. Observation of the thermal Casimir force. *Nat. Phys.* 2011, 7, 230-233. [CrossRef]
- [23] Bezerra, V.B.; Klimchitskaya, G.L.; Mohideen, U.; Mostepanenko, V.M.; Romero, C. Impact of surface imperfections on the Casimir force for lenses of centimeter-size curvature radii. *Phys. Rev. B* 2011, 83, 075417. [CrossRef]
- [24] Bordag, M.; Fischbach, E.; Klimchitskaya, G.L.; Krause, D.E.; Mostepanenko, V.M. Observation of the thermal Casimir force is open to question. *Int. J. Mod. Phys. A* 2011, 26, 3918-3929.
- [25] Chen, Y.-J.; Tham, W.K.; Krause, D.E.; López, D.; Fischbach, E.; Decca, R.S. Stronger limits on hypothetical Yukawa interactions in the 30–8000 nm range. *Phys. Rev. Lett.* 2016, 116, 221102. [CrossRef] [PubMed]
- [26] Behunin, R.O.; Intravaia, F.; Dalvit, D.A.R.; Maia Neto, P.A.; Reynaud, S. Modeling electrostatic patch effects in Casimir force measurements. *Phys. Rev. A* 2012, 85, 012504. [CrossRef]
- [27] Spreng, B.; Hartmann, M.; Henning, V.; Maia Neto, P.A.; Ingold, G.-L. Proximity force approximation and specular reflection: Application of the WKB limit of Mie scattering to the Casimir effect. *Phys. Rev. A* 2018, 97, 062504. [CrossRef]
- [28] Henning, V.; Spreng, B.; Hartmann, M.; Ingold, G.-L.; Maia Neto, P.A. Role of diffraction in the Casimir effect beyond the proximity force approximation. *J. Opt. Soc. Am. B* 2019, 36, C77-C87. [CrossRef]
- [29] Spreng, B.; Maia Neto, P.A.; Ingold, G.-L. Plane-wave approach to the exact van der Waals interaction between colloid particles. *J. Chem. Phys.* 2020, 153, 024115. [CrossRef]

- [30] Fosco, C.D.; Lombardo, F.C.; Mazzitelli, F.D. Proximity force approximation for the Casimir energy as a derivative expansion. *Phys. Rev. D* 2011, 84, 105031. [CrossRef]
- [31] Bimonte, G.; Emig, T.; Kardar, M. Material dependence of Casimir force: gradient expansion beyond proximity. *Appl. Phys. Lett.* 2012, 100, 074110. [CrossRef]
- [32] Bimonte, G.; Emig, T.; Jaffe, R.L.; Kardar, M. Casimir forces beyond the proximity force approximation. *Europhys. Lett.* 2012, 97, 50001. [CrossRef]
- [33] López, D.; Decca, R.S.; Fischbach, E.; Krause, D.E. MEMS-Based Force Sensor Design and Applications. *Bell Labs. Tech. J.* 2005, 10, 61–80. [CrossRef]
- [34] Lärmer F.; Schilp A. Method of anisotropically etching silicon. DE Patent 4241045, US Patent 5501893 and EP Patent 625285. 1996.
- [35] Kolb, P.W.; Decca, R.S.; Drew, H.D. Capacitive sensor for micropositioning in two dimensions. *Rev. Sci. Instrum.* 1998 69, 310–312. [CrossRef]
- [36] Simpson, W.; Leonhardt, U. (Eds.) *Forces of the Quantum Vacuum: An Introduction to Casimir Physics*; Chapter 4; World Scientific: Singapore, 2015.
- [37] Decca, R.S.; López, D. Measurement of the Casimir force using a microelectromechanical torsional oscillator: Electrostatic calibration. *Int. J. Mod. Phys. A* 2009, 24, 1748–1756. [CrossRef]
- [38] Chen, F.; Mohideen, U.; Klimchitskaya, G.L.; Mostepanenko, V.M. Experimental test for the conductivity properties from the Casimir force between metal and semiconductor. *Phys. Rev. A* 2006, 74, 022103. [CrossRef]
- [39] Behunin, R.O.; Dalvit, D.A.R.; Decca, R.S.; Genet, C.; Jung, I.W.; Lambrecht, A.; Liscio, A.; López, D.; Reynaud, S.; Schnoering, G.; et al. Kelvin probe force microscopy of metallic surfaces used in Casimir force measurements. *Phys. Rev. A* 2014, 90, 062115. [CrossRef]
- [40] Behunin, R.O.; Zeng, Y.; Dalvit, D.A.R.; Reynaud, S. Electrostatic patch effects in Casimir-force experiments performed in the sphere-plane geometry. *Phys. Rev. A* 2012, 86, 052509. [CrossRef]
- [41] Bulgac, A.; Magierski, P.; Wirzba, A. Scalar Casimir effect between Dirichlet spheres or a plate and a sphere. *Phys. Rev. D* 2006, 73, 025007. [CrossRef]
- [42] Emig, T.; Jaffe, R.L.; Kardar, M.; Scardicchio, A. Casimir Interaction between a Plate and a Cylinder. *Phys. Rev. Lett.* 2006, 96, 080403. [CrossRef]
- [43] Bordag, M. Casimir effect for a sphere and a cylinder in front of a plane and corrections to the proximity force theorem. *Phys. Rev. D* 2006, 73, 125018. [CrossRef]
- [44] Lambrecht, A.; Maia Neto, P.A.; Reynaud, S. The Casimir effect within scattering theory. *New J. Phys.* 2008, 78, 012115. [CrossRef]
- [45] Rahi, S.J.; Emig, T.; Graham, N.; Jaffe, R.L.; Kardar, M. Scattering theory approach to electromagnetic Casimir forces. *Phys. Rev. D* 2009, 80, 085021. [CrossRef]
- [46] Maia Neto, P.A.; Lambrecht, A.; Reynaud, S. Casimir energy between a plane and a sphere in electromagnetic vacuum. *Phys. Rev. A* 2008, 78, 012115. [CrossRef]
- [47] Emig, T. Fluctuation-induced quantum interactions between compact objects and a plane mirror. *J. Stat. Mech.* 2008, 2008, P04007. [CrossRef]
- [48] Canaguier-Durand, A.; Maia Neto, P.A.; Lambrecht, A.; Reynaud, S. Thermal Casimir Effect in the Plane-Sphere Geometry. *Phys. Rev. Lett.* 2010, 104, 040403. [CrossRef]
- [49] Hartmann, M.; Ingold, G.-L.; Maia Neto, P.A. Plasma versus Drude Modeling of the Casimir Force: Beyond the Proximity Force Approximation. *Phys. Rev. Lett.* 2017, 119, 043901. [CrossRef]
- [50] Bohren, C.F.; Huffman, D.R. *Absorption and Scattering of Light by Small Particles*; Chapt. 4; Wiley-VCH: Weinheim, Germany, 2004.
- [51] Olver, F.W.J.; Daalhuis, A.B.; Lozier, D.W.; Schneider, B.I.; Boisvert, R.F.; Clark, C.W.; Miller, B.R.; Saunders, B.V.; Cohl, H.S.; McClain, M.A. (Eds.) *NIST Digital Library of Mathematical Functions*. Release 1.1.1 of 2021-03-15. Available online: <http://dlmf.nist.gov/> (accessed on 6 April 2021).
- [52] Canaguier-Durand, A. *Multipolar Scattering Expansion for the Casimir Effect in the Sphere-Plane Geometry*. Ph.D. Thesis, Université Pierre et Marie Curie—Paris VI, Paris, France, 2011.
- [53] Schoger, T.; Ingold, G.-L. Classical Casimir free energy for two Drude spheres of arbitrary radii: A plane-wave approach. *arXiv* 2020, arXiv:2009.14090.
- [54] Bornemann, F. On the numerical evaluation of Fredholm determinants. *Math. Comp.* 2012, 79, 871–915. [CrossRef]
- [55] Boyd, J.P. Exponentially convergent Fourier-quadrature schemes on bounded and infinite intervals. *J. Sci. Comput.* 1987, 2, 99–109. [CrossRef]
- [56] Bordag, M.; Klimchitskaya, G.L.; Mohideen, U.; Mostepanenko, V.M. *Advances in the Casimir Effect*; Oxford University Press: Oxford, UK, 2015.

- [57] Fosco, C.D.; Lombardo, F.C. Mazzitelli, F.D. Derivative expansion for the Casimir effect at zero and finite temperature in  $d+1$  dimensions. *Phys. Rev. D* 2012, 86, 045021. [CrossRef]
- [58] Fosco, C.D.; Lombardo, F.C. Mazzitelli, F.D. Derivative-expansion approach to the interaction between close surfaces. *Phys. Rev. A* 2014, 89, 062120. [CrossRef]
- [59] Bimonte, G. Going beyond PFA: A precise formula for the sphere-plate Casimir force. *Europhys. Lett.* 2017, 118, 20002. [CrossRef]
- [60] Bimonte, G. Beyond-proximity-force-approximation Casimir force between two spheres at finite temperature. II. Plasma versus Drude modeling, grounded versus isolated spheres. *Phys. Rev. D* 2018, 98, 105004. [CrossRef]
- [61] Bimonte, G.; Emig, T. Exact Results for Classical Casimir Interactions: Dirichlet and Drude Model in the Sphere-Sphere and Sphere-Plane Geometry. *Phys. Rev. Lett.* 2012, 109, 160403. [CrossRef]
- [62] Canaguier-Durand, A.; Ingold, G.-L.; Jaekel, M.-T.; Lambrecht, A.; Maia Neto, P.A.; Reynaud, S. Classical Casimir interaction in the plane-sphere geometry. *Phys. Rev. A* 2012, 85, 052501. [CrossRef]
- [63] Bimonte, G. Classical Casimir interaction of a perfectly conducting sphere and plate. *Phys. Rev. D* 2017, 95, 065004. [CrossRef]
- [64] Rabinovich, S.G. *Measurement Errors and Uncertainties: Theory and Practice*; Springer: New York, NY, USA, 2000.
- [65] Launer, R.L.; Wilkinson, G.N. (Eds.) *Robustness in Statistics*; Academic Press: New York, NY, USA, 1979.
- [66] Klimchitskaya, G.L.; Mohideen, U.; Mostepanenko, V.M. The Casimir force between real materials: Experiment and theory. *Rev. Mod. Phys.* 2009, 81, 1827-1885. [CrossRef]
- [67] Klimchitskaya G.L.; Mostepanenko V.M. An alternative response to the off-shell quantum fluctuations: a step forward in resolution of the Casimir puzzle. *Eur. Phys. J. C* 2020, 80, 900. [CrossRef]