



Estimating a probability distribution corresponding to the negation of a property

Uyen Pham¹ · Ildar Batyrshin² · Nailya Kubysheva³ · Olga Kosheleva⁴

Accepted: 1 March 2021 / Published online: 5 April 2021

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Abstract

Suppose that we know the probability distribution corresponding to some property A . How can we then describe the distribution describing the negation of this property? In this paper, we show that, in general, we need additional information to uniquely determine the desired probability distribution. If we do not have this additional information and we still need to select one of the possible distributions for the negation, which one should we choose? We show that reasonable arguments lead to a formula proposed by Yager (IEEE Trans Fuzzy Syst 23(5):1899–1902, 2014). However, if we apply Yager’s formula to negation, we do not get the original distribution back. We show how to modify Yager’s formula so that the distribution for “not not A ” will be the same as the original distribution for the property A .

Keywords Probability distribution corresponding to negation · Fuzzy logic

1 Formulation of the problem

1.1 From probability of an event to probability of its negation: reminder

If we know the probability p of an event E , then we can determine the probability of its negation $\neg E$ (“not E ”) as $p' = 1 - p$.

If we apply this operation to the new value $p' = 1 - p$, we get the probability of $\neg\neg E$ —i.e., of the original event E :

this probability is equal to

$$1 - p' = 1 - (1 - p) = p.$$

1.2 What if we have a property instead of an event?

In many practical situations, we have a property A —e.g., a property of being a student. We may have students of different ages. For each possible age i , we can find the probability $p(i | A)$ that a randomly selected student is of age i . This probability can be found by dividing the number of all students of age i to the overall number of students.

These probabilities add up to 1:

$$\sum_i p(i | A) = 1.$$

In this sense, with respect to each quantity like age, each property A can be described by the corresponding probability distribution $p(i | A)$.

1.3 A natural question

For events, we can easily go from the probability of an event to the probability of its negation. So, a natural question is: we know the probability distribution corresponding to some property A ; what is the probability distribution $p(i | \neg A)$ corresponding to its negation $\neg A$?

Communicated by Vladik Kreinovich.

✉ Ildar Batyrshin
batyr1@gmail.com
Uyen Pham
uyenpham@uel.edu.vn
Nailya Kubysheva
aibolit70@mail.ru
Olga Kosheleva
olgak@utep.edu

¹ University of Economics and Law and John Von Neumann Institute, Ho-Chi-Minh City, Vietnam

² Centro de Investigación en Computación, Instituto Politécnico Nacional, Mexico City, Mexico

³ Kazan Federal University, Kazan, Russia

⁴ University of Texas at El Paso, El Paso, TX 79968, USA